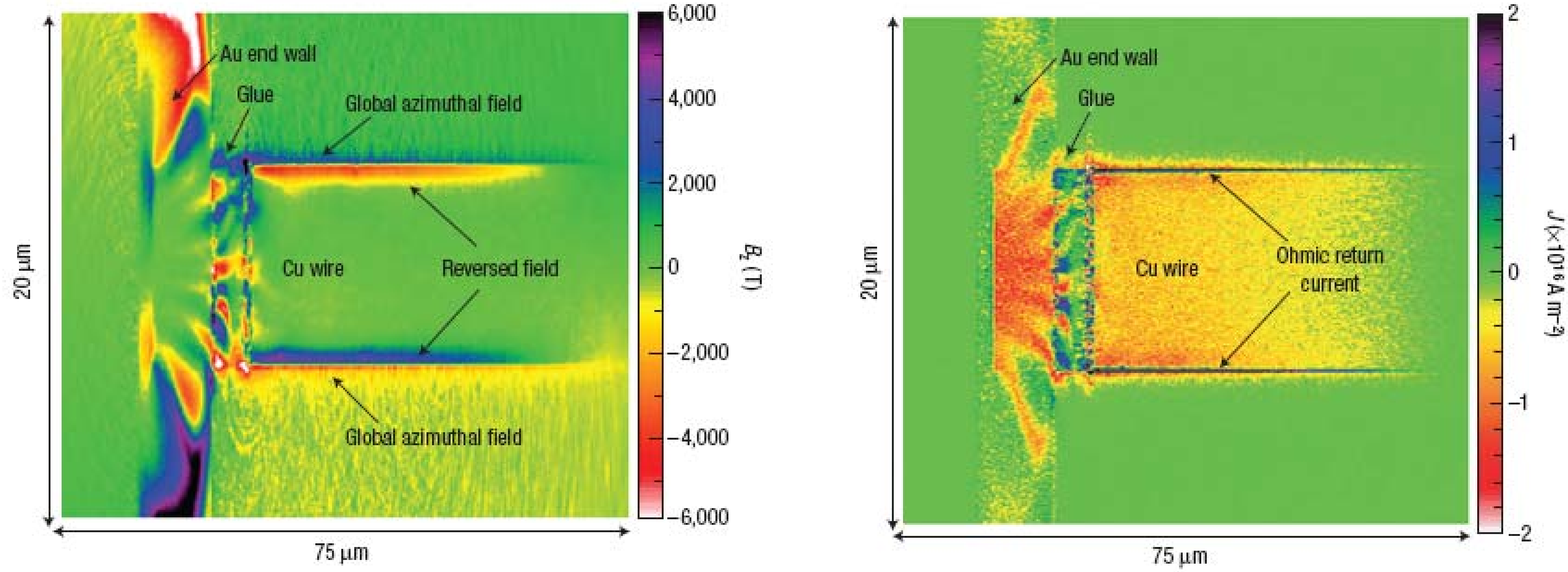
Hybrid Codes

A.P.L.Robinson
Central Laser Facility
STFC Rutherford-Appleton

A Standard Tool

Hybrid Codes : A **reduced** model that incorporates **most** relevant physics and runs **quickly**.



Excerpt from J.S.Green et al., *Nature Physics*, 2007
Illustrates many of the important aspects and appealing features of hybrid codes.

Content

- Definition
- Physics motivation for using hybrid codes.
- The Hybrid Approximation & Code Structure
- Resistivity
- What can we do with hybrid codes?
  - Fast Ignition
  - Cold Target Effects in Solid Target Interactions
  - Exploiting Resistivity

Preamble

Definition

- Term "hybrid code" or "hybrid model" crops up frequently in physics to mean many different things.
- Good example : hybrid code to a space physicist means "fluid electrons + kinetic ions"-type code
- In current laser-plasma studies we are particularly interested in the behaviour of "fast" (multi-MeV) electrons moving in a relatively cold dense plasma.
- Therefore we tend to mean the following:
  - Small population of very energetic electrons (kinetic)
  - Large background of cold electron (fluid)
  - Static or fluid ion background

Essence of Hybrid Codes

- Relativistic Electrons: Non-thermal, anisotropic, highly variable collisionality
- Cold Background: Highly collisional, near-Maxwellian, v.dense
- Collisions + Resistivity important: Resistive field generation & energy deposition
- Fast Tool for Large Scale Problem

→ Need a kinetic component with collisions

→ Want to just treat as a fluid

→ Need to incorporate resistivity and collisional energy deposition

→ Must run quickly and be able to solve full problem

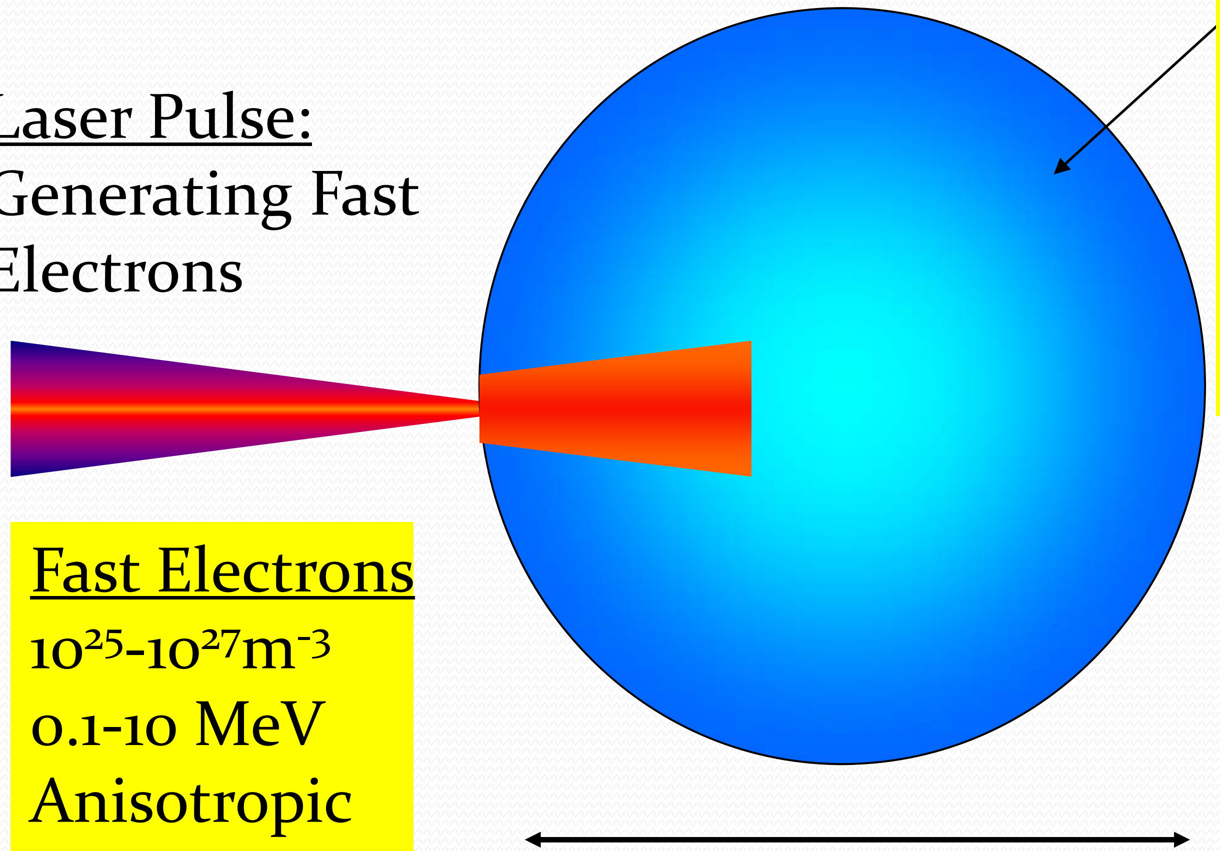
Physical Motivation

Main Justification for Hybrid Codes

- Full calculations (e.g. via collisional PIC) of fast electron transport through dense plasma require very large computational resources.
- Sacrifice some degree of accuracy to produce a "reduced model" that will run quickly and allow one to carry out useful calculations.

General Situation of Interest – A Sketch

Laser Pulse:
Generating Fast Electrons



Dense Plasma
1g/cc – 500g/cc
10^{29}-10^{32}m^{-3}
0-100eV
Maxwellian or Degenerate

Fast Electrons
10^{25}-10^{27}m^{-3}
0.1-10 MeV
Anisotropic
Non-thermal

100s of µm

Time of interest
1-30ps

Physical Motivation for Hybrid Codes

- <u>Fast Electron Physics</u>
  - Small population
  - Low collision rate
  - Non-thermal / Anisotropic
  - Treatment by fluid equations not sufficient
  - Must treat kinetically

$$\alpha I = n_f v_f T_f$$

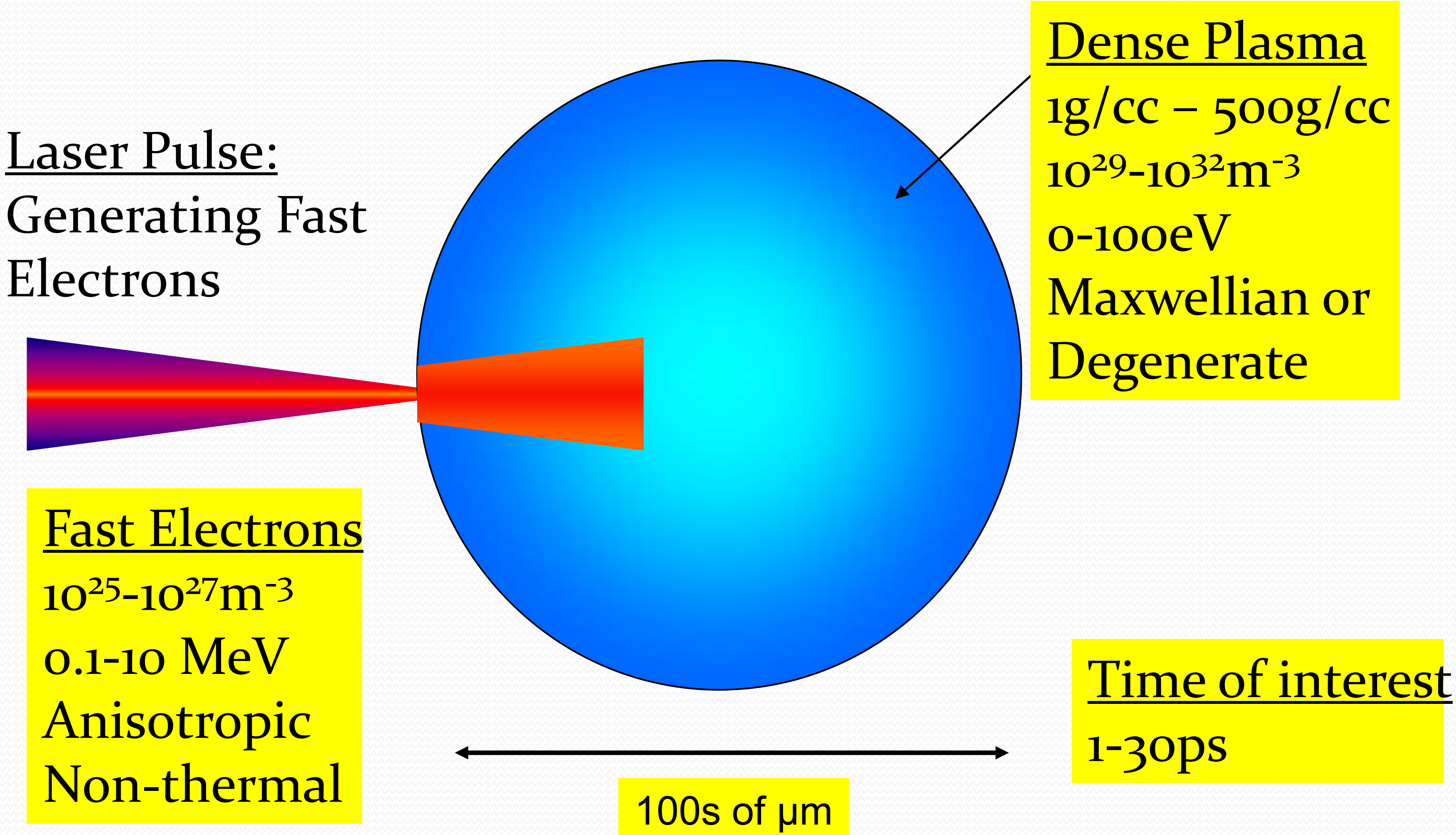
$I \sim 10^{20}$ Wcm^{-2}
v ~ c
T ~ 0.3-10MeV
α ~0.1-0.5
So
$n \sim 10^{20}$-10^{21} cm^{-3}
n ~ 0.1-1 n_c

Typical fast electron scattering time assuming Beg's Law scaling.

$$\tau = \frac{60}{Z n_{23}} \sqrt{I_{18} \lambda^2_{\mu m}} \quad \text{psec}$$

$I \sim 10^{18}$ Wcm^{-2} (1 micron wavelength)
$n_c \sim 10^{23}$cm^{-3} ; Z = 13 (Al)
t ~ 4.6ps

General Situation of Interest – A Sketch



Physical Motivation for Hybrid Codes

- Cold Electron Physics:
  - The bulk population
  - ($n_c \sim 10^{29}$-10^{32}m^{-3})
  - Relatively low energy
  - ($T_c \sim$ 1eV – 10keV)
  - Highly collisional (and resistive)
  - Fairly close to equilibrium distribution (e.g. Maxwellian)
  - Degeneracy and quantum effects may be important.
  - Want to treat as a fluid!

$$\tau_{ei} = 0.028 \frac{4}{Z}\left(\frac{T_c}{keV}\right)^{3/2}\left(\frac{10}{\ln\Lambda}\right)\left(\frac{n_c}{10^{22}\,cm^{-3}}\right)^{-1} \text{ psec}$$

$T_c \sim$ 1keV
$n_c \sim 10^{23}$cm^{-3} ; Z = 13 (Al)
$\ln\Lambda \sim 10$
$t \sim$ 0.86fs

Compressed Fast Ignition fuel
At 10^{26}cm^{-3}

$$E_F = \frac{\hbar^2}{2m_e}\left(3\pi^2 n_c\right)^{2/3}$$

E ~ 200eV in compressed fuel

General Situation of Interest – A Sketch

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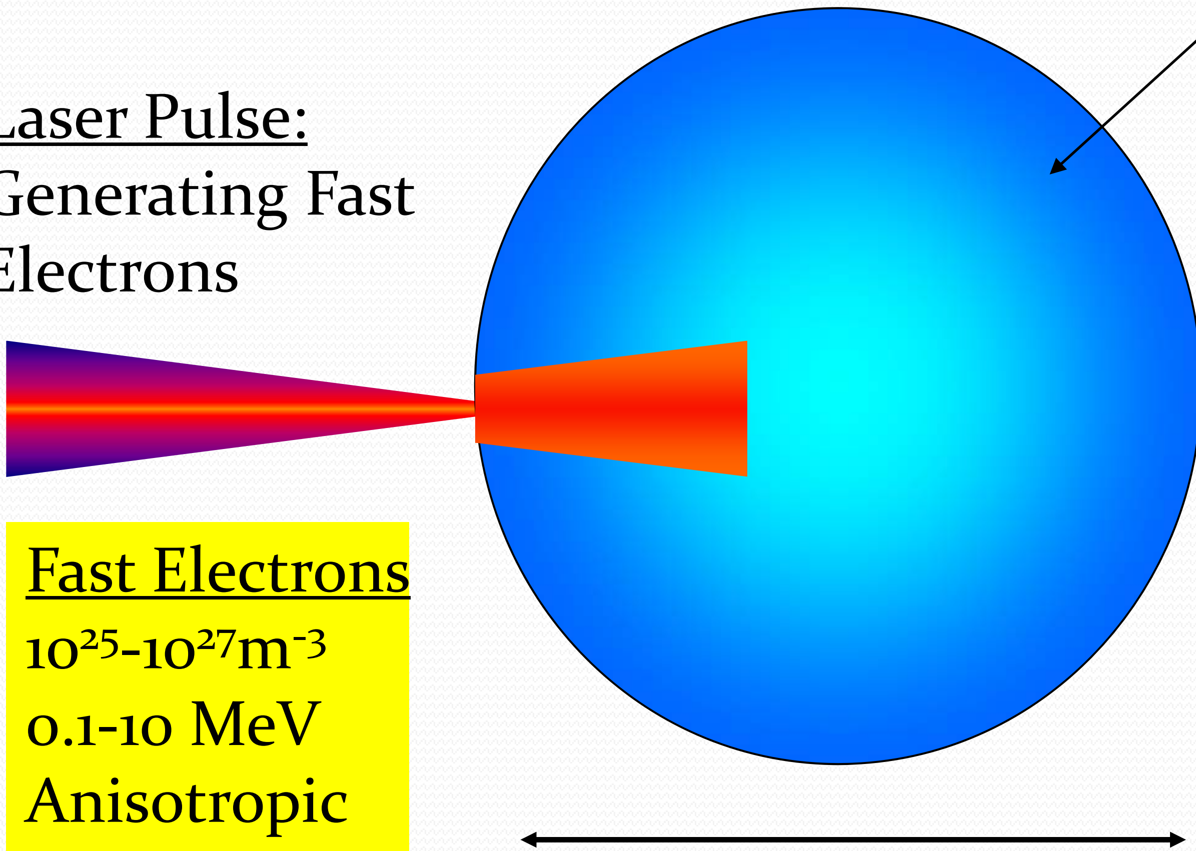

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Physical Motivation for Hybrid Codes

- Scales:
  - Hydrodynamic:
    - Fuel dis-assembly time ~ 10ps
    - Foil decompression time > 10ps
  - Cold electrons:
    - Cold electron Debye length < 1nm
    - Cold electron plasma period < 1fs
  - Fast electrons:
    - Fast electron propagation distance > 100 μm
    - Fast electron Debye length ~1μm
    - Fast electron pulse duration ~ 1-10ps

Fully kinetic needs to resolve these scales

Hybrid only needs to resolve these scales

Hard to deal easily with such disparate scales!!

Rudimentary Electrodynamics

- The "gist":
- Basic 1D model:

$$n_c u_c = n_f u_f$$

$$\frac{n_f}{n_c} = 0.001 - 0.01$$

$$u_c \approx 10^5 - 10^6 \,\mathrm{ms}^{-1}$$

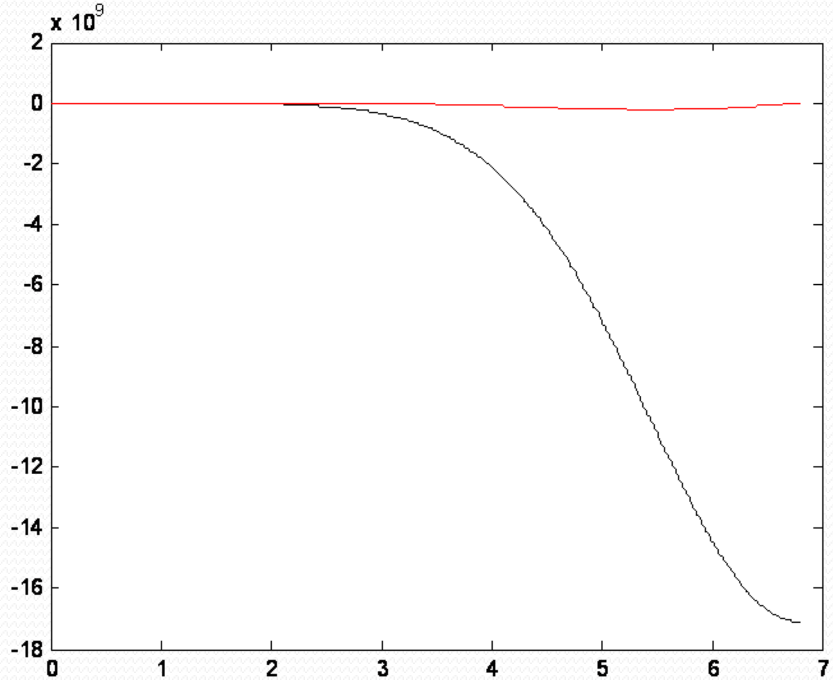
$$\frac{\partial u_c}{\partial t} = -\frac{eE}{m_e} - \frac{u_c}{\tau}$$

$$\frac{\partial E}{\partial t} = \frac{e n_c u_c}{\varepsilon} + \frac{e n_f u_f}{\varepsilon}$$

$$z = x - u_f t$$

$$\frac{e^2 n_c}{\varepsilon m_e u_f^2}$$

$$\frac{c}{\omega_{p,c}} = O(10nm)$$



Physical Motivation for Hybrid Codes

- <u>Role of Resistivity I:</u>
  - Fast electron current density is $> 10^{14}\mathrm{Am}^{-2}$
  - Must be balanced by a *local* return current to a good approximation
  - In collisionless plasma the electric field required for this will be tiny.
  - Not interested in small fluctuations.
  - In a resistive plasma it will be significant.
  - <u>Thus resistive inhibition is possible</u>.

Spitzer Resistivity

$$\eta \cong 10^{-4} \frac{Z \ln \Lambda}{T_{c,eV}^{3/2}}$$

Z=13, T = 100eV, $\eta \sim 10^{-6}\Omega\mathrm{m}$

$$\mathbf{j_f} + \mathbf{j_c} \approx 0$$

$$\mathbf{E} \approx -\eta \mathbf{j}_f$$

For $10^{16}\mathrm{Am}^{-2}$
$E \sim 10^{10}\mathrm{Vm}^{-1}$

i.e. 1MeV in 100 microns

Physical Motivation for Hybrid Codes

- <u>Role of Resistivity II:</u>
  - Since curl of electric field is not zero, B field will be grown.
  - Typical rate of field growth can be estimated from Faraday's Law
  - 100T fields with spatial sizes on the micron scale are sufficient to strongly deflect and guide electrons
  - <u>Therefore magnetic collimation and filamentation are distinct possibilities</u> →

$$\frac{\partial \mathbf{B}}{\partial t} = -\nabla \times \mathbf{E}$$

$$\frac{B}{t} \approx \frac{\eta j_f}{L}$$

t ~ 1ps ; $\eta \sim 10^{-6}$
$j \sim 10^{16} Am^{-2}$
L ~ 10 μm
So…
B ~ 1000T

Physical Motivation for Hybrid Codes

- <u>Role of Resistivity III:</u>
  - Resistivity also means that Ohmic heating will be strong.
  - Heating to 100s of eV is possible.
  - <u>Therefore solid targets will be heated significantly by Ohmic mechanism.</u>
  - In compressed ICF fuel the fast electrons are stopped quickly by <u>collisional drag</u> instead of Ohmic heating.

$$\frac{\partial T_{c,eV}}{\partial t} = \frac{\eta j^2}{eCn_c}$$

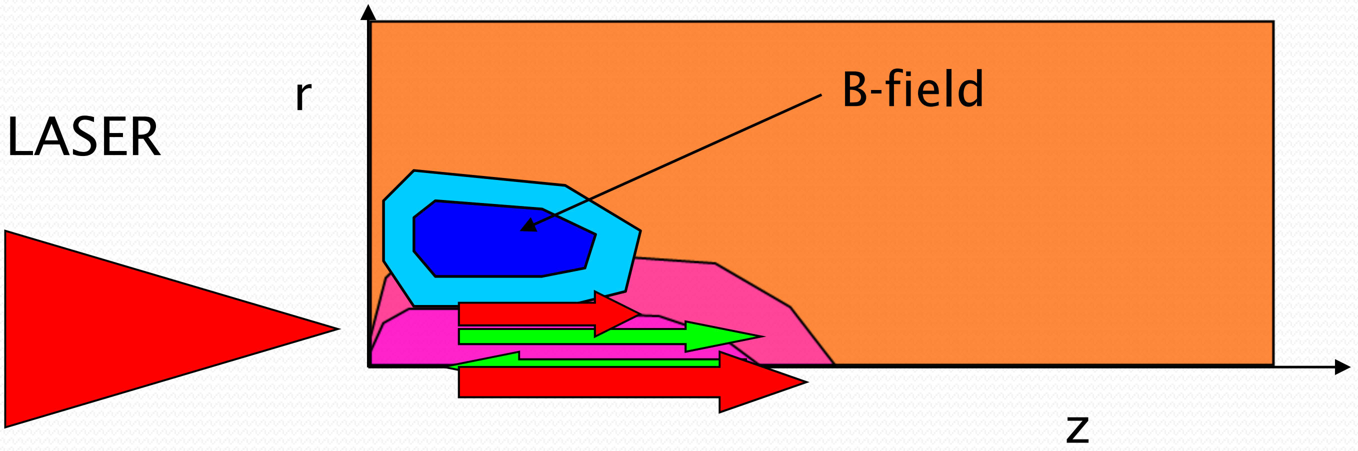
$$\frac{\partial T_{c,eV}}{\partial t} \approx 10\text{eV/fs}$$

$\eta \sim 10^{-6}$

$j \sim 10^{16}\text{Am}^{-2}$

$n_c \sim 10^{23}\text{cm}^{-3}$

Resistive Fields + Heating



Physical Motivation for Hybrid Codes

- So we can simplify numerical simulation if we
  - Treat only the fast electron kinetically
  - Treat the bulk plasma as a fluid
- This allows us to
  - Use much larger cell sizes and timesteps
  - Reduce Maxwell's Equations
  - Include resistivity more easily
  - Easily include cold target physics
- This depends on the previous arguments amounting to being **good approximations.**
- Hybrid codes do rely on **approximations**.

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Hybrid Approximation and Code Structure

Key features

- Kinetic Treatment of Fast Electrons
- Background Plasma treated as a resistive fluid
- Use reduced Maxwell's equations

Fast electrons

- Fully kinetic
- Include E and B (which will be resistively generated)
- Scattering and Drag included (particularly important at high density)

$$\frac{\partial f}{\partial t} + \mathbf{v}.\frac{\partial f}{\partial \mathbf{r}} - e\left(\mathbf{E} + \mathbf{v} \times \mathbf{B}\right).\frac{\partial f}{\partial \mathbf{p}} = C(f)$$

Resistive Fields

Scatter and Drag

- Effectively solve the Kinetic (or Vlasov) equation by using Particle-in-Cell methods

Background Plasma I

- Background is a fluid. Describe by density, velocity, and internal energy.
- Can treat this as a static fluid *if* hydrodynamic motion can be neglected.
- In this case background has a fixed density, but T will vary (so electron density and Z can thus vary as well).

In general = Full hydrodynamic/fluid description

Minimal set

$$\frac{\partial T_{c,eV}}{\partial t} = \frac{\eta j^2}{eCn_c} + D_f$$

Heating of background by Ohmic Heating + Drag on Fasts

$$\eta = \eta(T)$$

Resistivity (Temperature dependent)

Background Plasma II

- If hydrodynamic motion is important then we need to evolve the background via MHD equations
- So the standard considerations for a hydrocode apply to this (recall hydro lecture!)
- Electron-Ion energy exchange and radiative cooling might be important on these time-scales
- Nonetheless the resistivity is usually the most important aspect of the background plasma to consider.
- In high temperature weakly coupled plasmas one can use the Spitzer resistivity
- Otherwise accurately determing the resistivity + background atomic model in general can be difficult.

$$\eta = ?$$

Reduction of Maxwell's Equations I

- Key assumption is that of current balance.
- Also note that electron pulse durations are bigger than characteristic light transit time.
- Pulse duration = τ
- Beam width = L

neglect

$$\frac{\partial \mathbf{E}}{\partial t} = -\frac{\mathbf{j}}{\varepsilon_0} + c^2 \nabla \times \mathbf{B}$$

$$\mathbf{j_f} + \mathbf{j_c} = \frac{1}{\mu_0} \nabla \times \mathbf{B}$$

Dimensional Analysis Argument:

$$\frac{\left|\frac{\partial E}{\partial t}\right|}{\left|c^2 \nabla \times B\right|} \approx \frac{\frac{\eta j}{\tau}}{\frac{c^2 \eta j \tau}{L^2}} \approx \frac{L^2}{c^2 \tau^2}$$

So it is valid if L<< cτ , e.g. <u>Even if </u> 5 micron beam radius, and 50fs pulse duration Ratio is still 1/9

Reduction of Maxwell's Equations II

- <u>Current Balance</u>
  - Formally we have:

$$\mathbf{j_f} + \mathbf{j_c} = \frac{1}{\mu_0}\nabla \times \mathbf{B}$$

  - Actually to a good approximation:

$$\mathbf{j}_f + \mathbf{j}_c \approx 0$$

- Electrostatic Argument

$$\frac{\partial E}{\partial t} = -\frac{j}{\varepsilon_0} \Rightarrow \frac{\partial E}{\partial t} \approx 10^{12}\,\mathrm{Vm}^{-1} fs^{-1}$$

- Magnetostatic Argument:
  - See A.R.Bell et al. PPCF **48**, R37 (2006)
  - If cancellation is not local then this would permit growth of B-fields with energy in excess of beam energy.
  - So cancellation must be *local* as well.

Reduction of Maxwell's Equations III

- We then determine the electric field through an Ohm's Law:

$$\mathbf{E} = \eta \mathbf{j}_c$$

$$\boxed{\mathbf{E} = -\eta \mathbf{j_f} + \eta c^2 \nabla \times \mathbf{B}}$$

Reduction of Maxwell's Equations II

- Finally we use this Ohm's Law in Maxwell's Induction Equation to evolve the magnetic field.

$$\frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\eta \mathbf{j_f}) + \frac{\eta}{\mu_0} \nabla^2 \mathbf{B}$$

Hybrid Code Architecture

Interpolate to grid to get current density

Determine E and Evolve B

Interpolate Fields to Fast Electrons

Push Fast Electrons, i.e. Update momentum

Apply Angular Scattering and Drag

Heat Background

Update Resistivity

Solve Hydro?

Electron Ion exchange?
Radiation ?
Etc.

Inject Fast Electrons

Move Fast electrons

Vlasov Solution via PIC

Solve Fluid Background

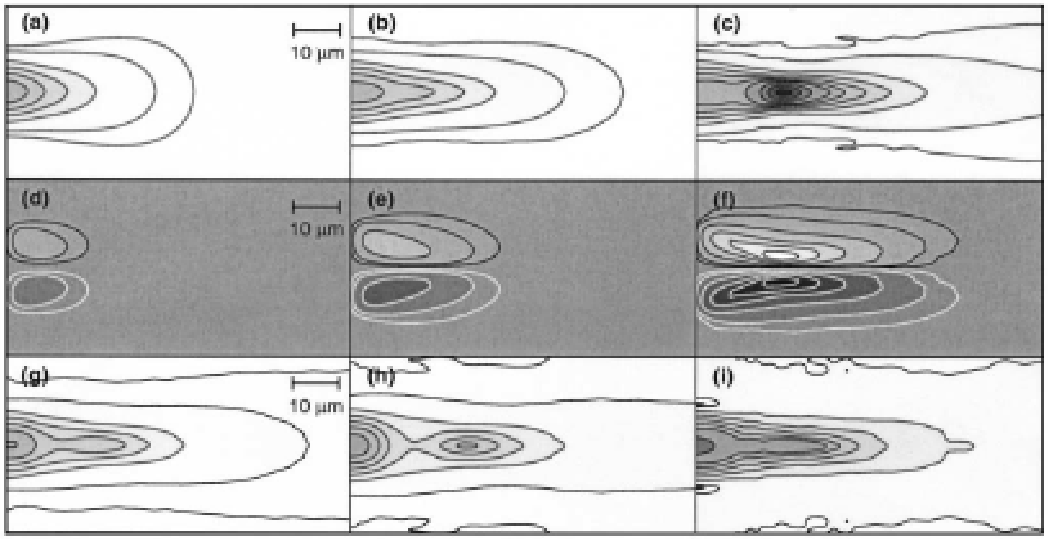
Reduce Maxwell's Equations

Hybrid Code Resource Requirements

- Hybrid code can easily run in 2D or 3D on a modern desktop
- 10M particles as the fasts (~ 1GB) + Similar for gridded quantities will suffice for a 3D calculation.
- Comparable calculation with collisional PIC would require a cluster computing resource + a longer run time.
- Key to why we use hybrid codes:
- Sacrifice some degree of accuracy to produce a "reduced model" that will run quickly and allow one to carry out useful calculations.

Do we retain accuracy?

- Bell and Kingham PRL 2003
- Showed results from full finite-difference Vlasov-Fokker-Planck calculations with full Maxwell's equations.
- Results comparable to hybrid results.



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Resistivity

Resistivity in Plasmas

- The most commonly referenced form of plasma resistivity is the Spitzer resistivity:

$$\eta \cong 10^{-4} \frac{Z \ln \Lambda}{T_{e,eV}^{3/2}}$$

- Derive this by linearizing the Vlasov-Fokker-Planck equation.
- Primarily depends on T and Z
- Valid in the weakly-coupled limit

Resistivity in Solids and Dense Plasmas

- In laser-solid problems we will have dense, relatively low temperature plasmas
- Will not be weakly coupled and may be degenerate
- Spitzer resistivity would imply an collision time shorter than the electron transit time between ions.

Solid Al

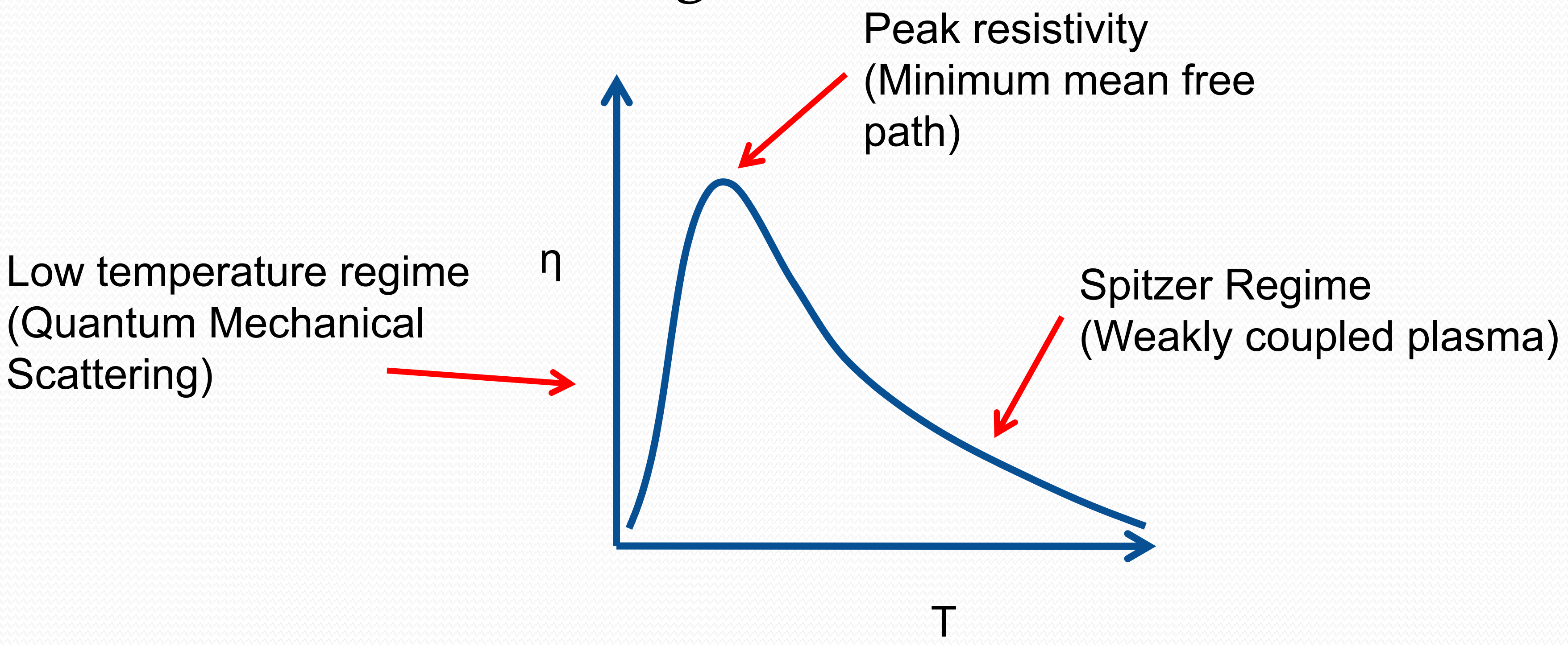
Interatomic spacing $$r_s = \left(\frac{3}{4\pi n_i}\right)^{1/3}$$ $$r_s = 1.6\times 10^{-10}\,\mathrm{m}$$

Transit time of a 10eV electron is $$\tau_{\min} = \frac{r_s}{\sqrt{\frac{2eT_{e,eV}}{m_e}}} = 0.084\mathrm{fs}$$

Spitzer collision period is $$\tau_{ei} = 0.028\frac{4}{Z}\left(\frac{T_c}{keV}\right)^{3/2}\left(\frac{10}{\ln\Lambda}\right)\left(\frac{n_c}{10^{22}\,cm^{-3}}\right)^{-1}\mathrm{ps} = 0.007\mathrm{fs}$$

Resistivity in Solids and Dense Plasmas II

- The implication is therefore that the "resistivity curve" must look something like:



What is the resistivity curve?

- Dense Plasma theorists will use Quantum Molecular Dynamics – Density Functional Theory (QMD-DFT) calculations to determine curve up to a few eV.
- QMD-DFT treats ions classically but the electrons are solved in a fully quantum mechanical way using DFT.
- Moderate to high temperatures are covered by models such as the Lee-More-Desjarlais model
- Need to fold in degeneracy, ionization and scattering rate to get accurate result.

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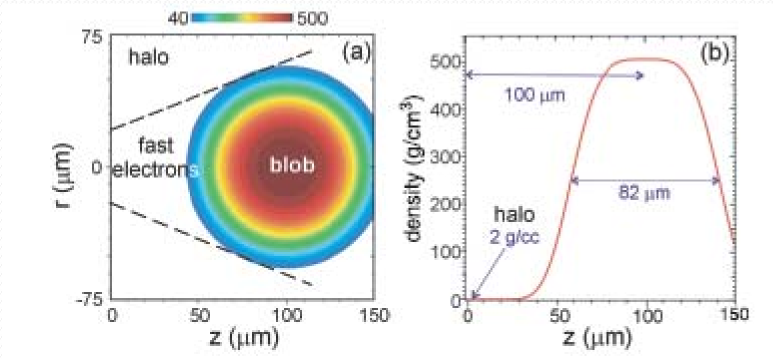
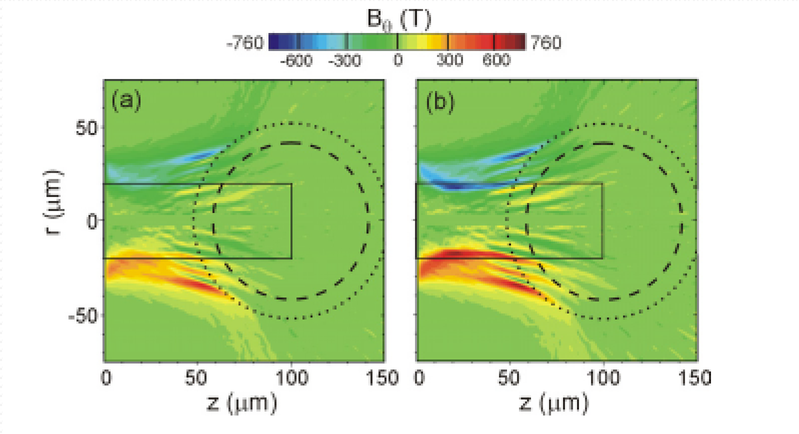
Applications of Hybrid Codes

Fast Ignition

- Density is so high in compressed DT fuel that application of conventional PIC codes to full problem is essentially impossible.
- Use of hybrid methods is essential.
- Hybrid important for addressing basic questions such as : how much ignitor pulse energy is required?

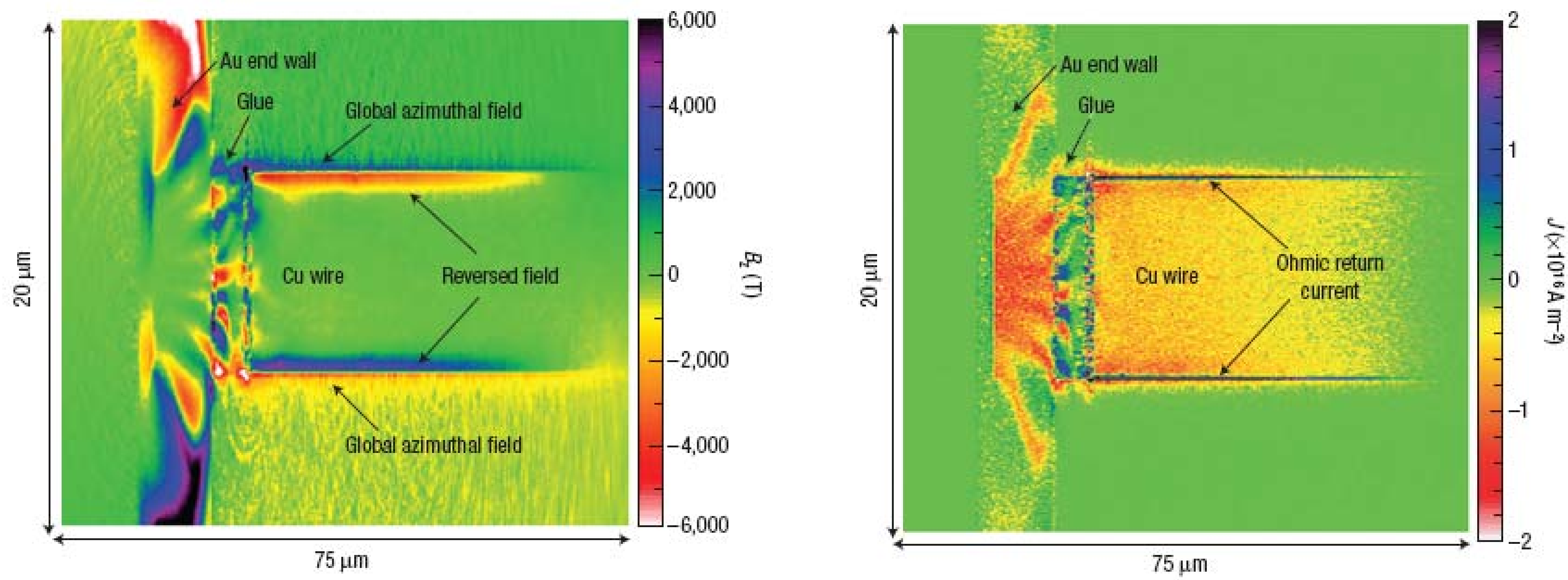




Excerpts from J.Honrubia, PPCF, **51**, 014008 (2009)

Solid Targets

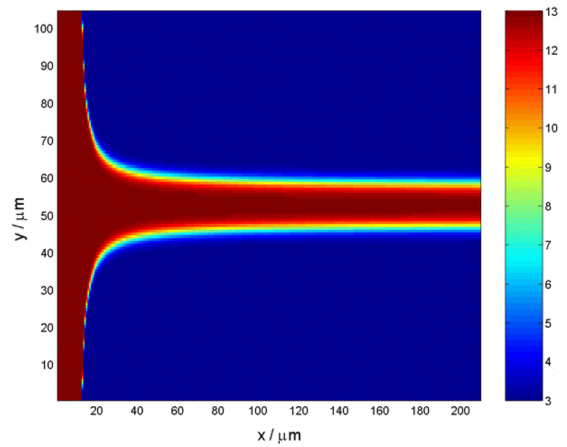
- Standard tool in laser-solid target simulation.
- Many experiments carried out in this regime.
- Allows rapid analysis of experimental results to provide interpretation.

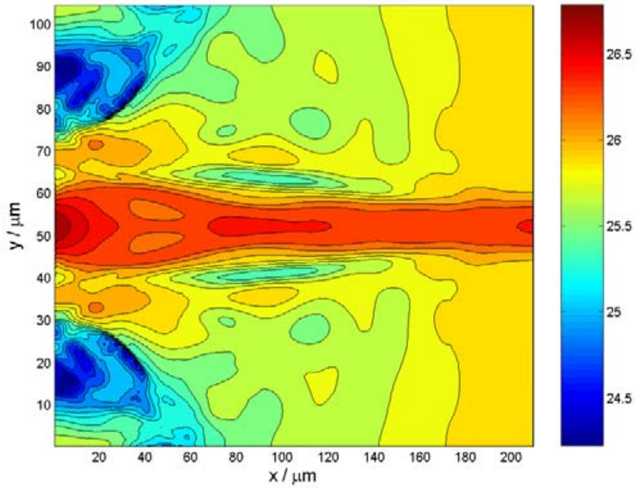


Excerpt from J.S.Green et al., *Nature Physics*, 2007

Studying New Ideas

- Hybrid codes are useful for studying new concepts as well.
- Particularly where we directly exploit the physics contained in hybrid codes, e.g. Look at exploiting resistivity gradients.





Excerpts from Robinson and Sherlock, PoP 2007

Summary I

Sacrifice some degree of accuracy to produce a "reduced model" that will run quickly and allow one to carry out useful calculations.

- Physics based motivation for using hybrid codes.
- Approximations, consituent equations, and structure of a hybrid code.
- Problems that hybrid codes have been used to address.

Summary II

- Relativistic Electrons: Non-thermal, anisotropic, highly variable collisionality
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