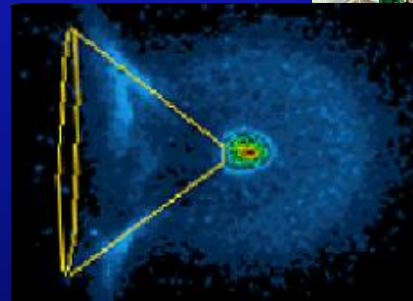
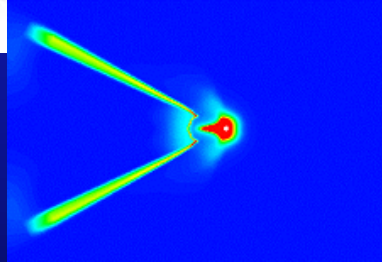


Hydrodynamic Simulations

- Physics, methods and codes -

- Radiation Hydrodynamic Simulations
- Simulation examples for Fast Ignition and high energy density physics



4th UK-Japan Winter School in High Energy Density Science
January 5, 2011, Edinburgh, The Royal Observatory

Institute of Laser Engineering, Osaka University

Hideo Nagatomo

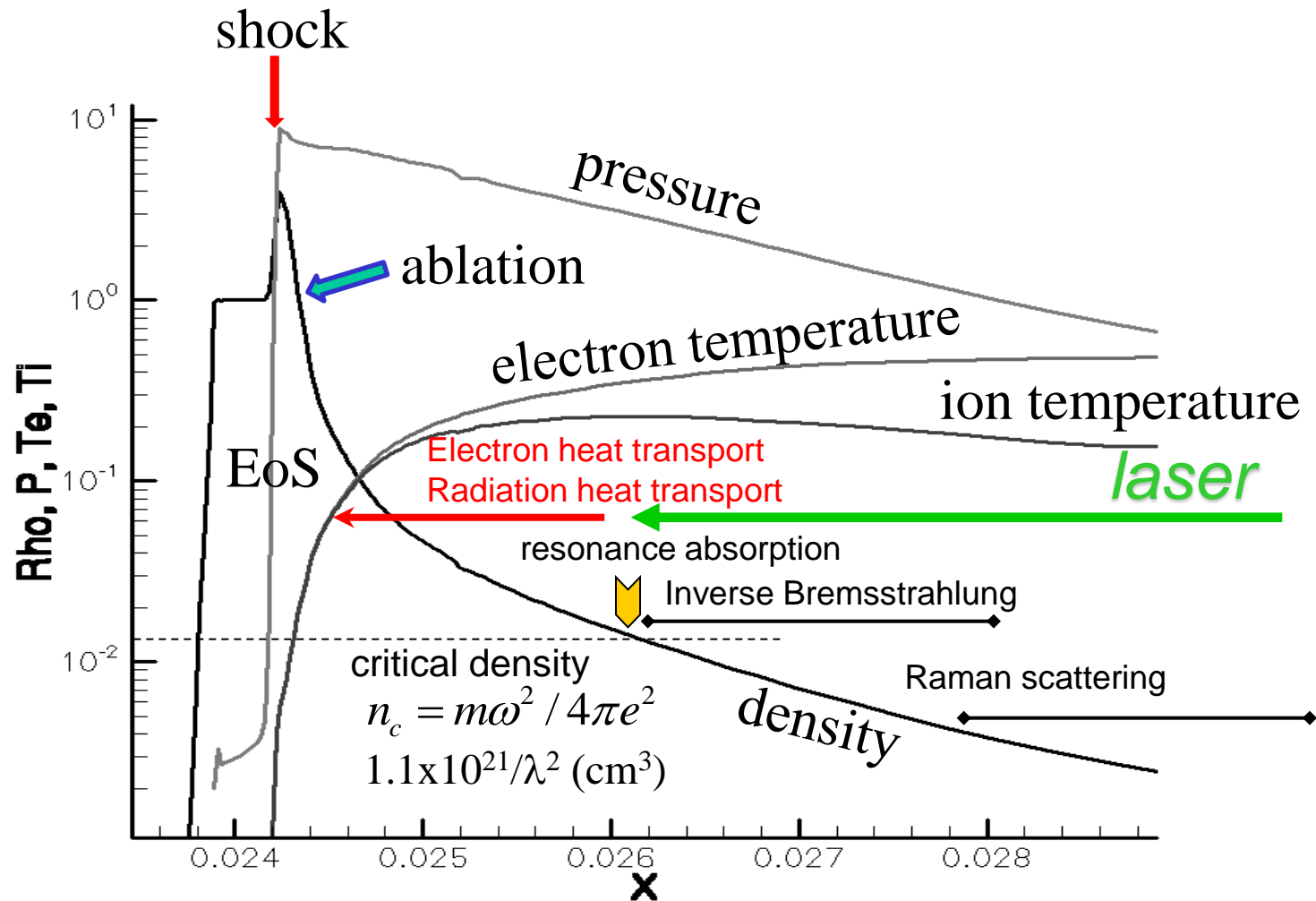


Contents

- Numerical method for radiation hydrodynamics
 - conservation, non-conservation
 - Arbitrary Lagrangian Eulerian method
- Recent computational simulations related to fast ignition
 - Radiation hydrodynamics
 - Fast ignition integrated interconnecting simulation system (FI³)



density, temperature and pressure profiles of laser absorption and ablation



$$\mathbf{v}_g \cdot \nabla \mathbf{I}_L^k = -\mathbf{v}_{\text{abs}} \mathbf{I}_L^k \quad \mathbf{S}_L = \sum_k \mathbf{v}_{\text{abs}} \mathbf{I}_L^k / \mathbf{v}_g^k$$

Basic equation system of hydrodynamic



• Mass

$$\frac{\partial \rho}{\partial t} + \frac{\partial(\rho u)}{\partial x} = 0$$

• Momentum

$$\frac{d\rho}{dt} = \frac{\partial \rho}{\partial t} + u \frac{\partial \rho}{\partial x} = -\rho \frac{\partial u}{\partial x}$$

• Energy

• electron energy

$$\frac{\partial}{\partial t}(\rho u) + \frac{\partial}{\partial x}(\rho u^2 + p) = 0$$

• ion energy

$$\rho \frac{du}{dt} = \rho \left(\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} \right) = -\frac{\partial p}{\partial x}$$

• Thermal transport

• Radiation transport

• Equation of state

$$\frac{\partial}{\partial t}(e) + \frac{\partial}{\partial x}(eu + pu) = 0$$

$$e = \rho \left(\varepsilon + \frac{1}{2} u^2 \right)$$

$$p = (\gamma - 1)\varepsilon$$

$$\rho \frac{d\varepsilon}{dt} = \rho \left(\frac{\partial \varepsilon}{\partial t} + u \frac{\partial \varepsilon}{\partial x} \right) = -p \frac{\partial u}{\partial x}$$

• Laser ray-tracing

• Laser absorption

non-conservation form

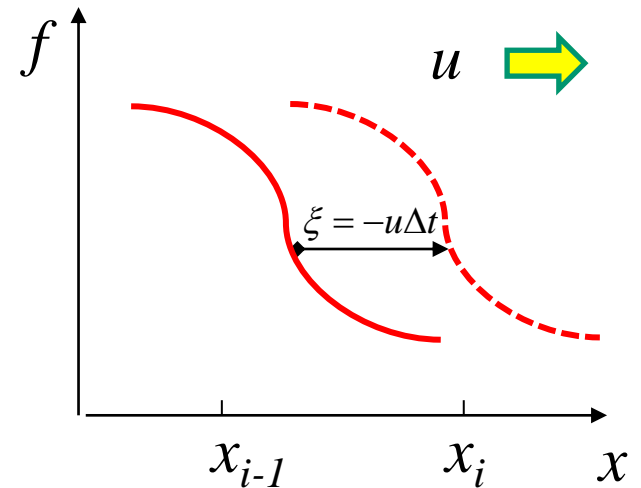
New ALE Method with CIP

1. Rezone the mesh for suitable shape
2. Obtain new value by solving the equation using CIP
(Constrained Interpolation Profile ;Cubic-Interpolated Pseudo-Particles: Yabe *et. al.* JCP, 169, 556-593, 2001)

$$\frac{\partial f}{\partial t} + u \frac{\partial f}{\partial x} = 0 \quad \frac{\partial f'}{\partial t} + u \frac{\partial f'}{\partial x} = -f' \frac{\partial u}{\partial x}$$

$$f_i^{n+1} = a_i \xi^3 + b_i \xi^2 + c_i \xi + d_i \quad \xi = -u \Delta t$$

$$f_i'^{n+1} = 3a_i \xi^2 + 3b_i \xi + c_i$$



CIP method has high accuracy.

- Less dissipative.
- easy to make a code.
- Stable and robust. C-CUP, RCIP



ALE

$$u = u_0 - u_g$$

rezoning; track mass center

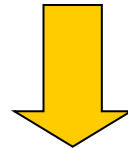
Classical ALE and ALE-CIP method

1. solve the Lagrangian method
2. arrange the computational mesh (U_{ALE} is obtained)

$$\underbrace{\frac{\partial f}{\partial t} + U \frac{\partial f}{\partial x}}_{\frac{df}{dt}} + (U_{ALE} - U) \frac{\partial f}{\partial x} = 0$$

f : variable
 U : fluid velocity
 U_{ALE} : grid node velocity

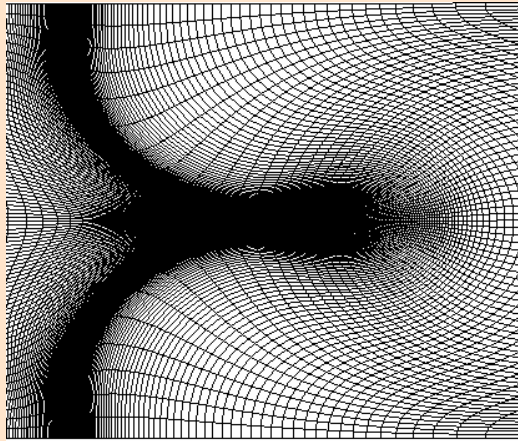
cf. "An Arbitrary Lagrangian-Eulerian Computing Method for All Flow Speed", C.W. Hirt, A.A. Amsden, and J.L. Cook, JCP 14, 227-253 (1974).



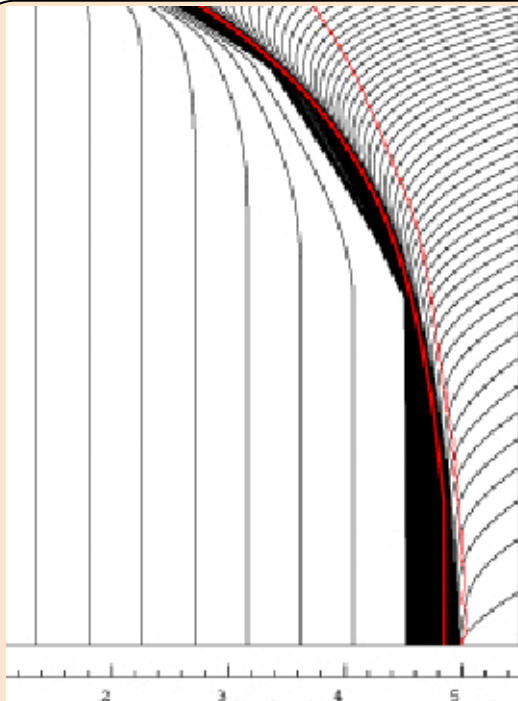
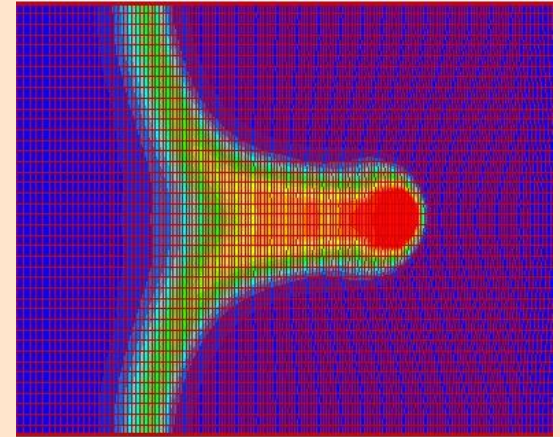
1. grid velocity is included in CIP method.

$$\frac{\partial f}{\partial t} + U_{ALE} \frac{\partial f}{\partial x} = 0$$

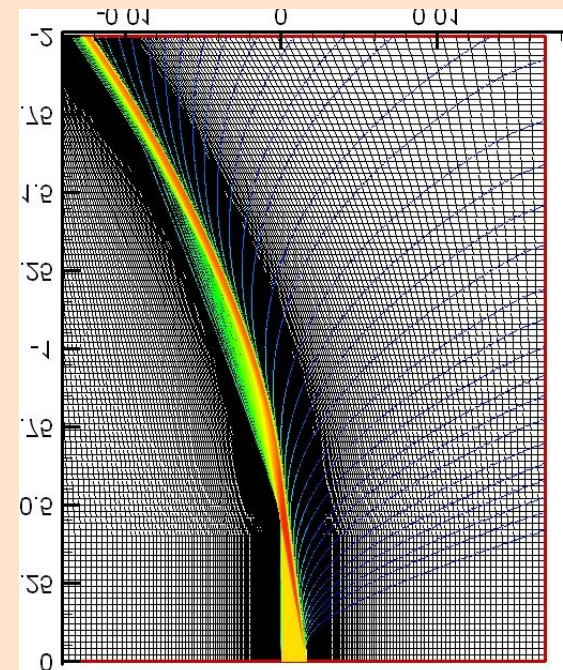
Conventional ALE and ALE-CIP




- difficulty of rezoning/
remapping in C-ALE
- difficulty of solving diffusion
equations in C-ALE
- material tracking is easier.



- rough computational mesh
at laser absorption region in
C-ALE.



Electron heat conduction

energy density	U			
average velocity	V_{av}		heat flux	$q = -\frac{V_{av} l}{3} \nabla U$
mean free path	l			

$$V_{av} = \sqrt{k_B T_e / m}$$



$$q = -\kappa_0 T_e^{5/2} \nabla T$$

$$l_e = 1/(n_e \sigma_c) \propto T_e^2$$

diffusion approximation ; Spitzer-Härm (1962) thermal conductivity

$$\kappa_e = \kappa_0 T_e^{5/2} = (8/\pi)^{3/2} G(Z) \frac{(k_B T_e)^{5/2} k_B}{Z e^4 m_e^{1/2} \ln \Lambda}$$

Z dependance ; $G(Z) \approx (1 + 3.3/Z)$

Coulomb logarithms ; $\ln \Lambda$

heat transport

- Classical heat transport

- diffusion approximation $q_{SH} \propto T_e^{5/2} \nabla T_e$
Braginskii's thermal conductivity

- flux limited diffusion $q_{effective}^{-1} = q_{SH}^{-1} + q_L^{-1}$ $q_L = f_L n_e v_e T_e$

- Kinetic effect
(non-local heat transport)

- Fokker-Planck equation

$$\frac{\partial f}{\partial t} + \mathbf{v} \cdot \nabla f - \frac{e(\mathbf{E} + \mathbf{V} \times \mathbf{B})}{m_e} = C(f) + H(f)$$

➔ Legendre polynomials

multi-group diffusion-type equations for radiation transport



frequency: ν intensity: I^ν emissivity: η^ν opacity: χ^ν

Fundamental equation for radiation transport (6-D Boltzmann equation)

$$\frac{1}{c} \frac{\partial I^\nu}{\partial t} + \Omega \cdot \nabla I^\nu = \eta^\nu - \chi^\nu I^\nu$$

Legendre expansion 0-th order momentum equation is ;

$$\frac{1}{c} \frac{\partial E^\nu}{\partial t} + \nabla \cdot \mathbf{F}^\nu = 4\pi\eta^\nu - \chi^\nu c E^\nu$$

1st order momentum equation is ;

$$\frac{1}{c} \frac{\partial \mathbf{F}^\nu}{\partial t} + c \nabla \cdot \mathbf{P}^\nu = -\chi^\nu c \mathbf{F}^\nu$$

under isotropic assumption, and $c \gg 1$, finally ; $\mathbf{P}^\nu = \frac{1}{3} E^\nu$ $\frac{c}{3} \nabla E^\nu = -\chi^\nu \mathbf{F}^\nu$

$$\nabla \cdot \left(-\frac{c}{3\chi^\nu} \nabla E^\nu \right) = 4\pi\eta^\nu - \chi^\nu c E^\nu \quad \text{anisotropic case ; Eddington's factor } f^\nu$$

$$\mathbf{F}^\nu = -\frac{c}{\chi^\nu} \nabla \cdot (f^\nu E^\nu)$$

Fundamental equations of radiation hydrodynamics for laser plasma simulation

(1 fluid, 2 temperature plasma)

(mass)

$$\frac{d\rho}{dt} = -\rho \nabla \cdot \mathbf{u}$$

(momentum)

$$\rho \frac{d\mathbf{u}}{dt} = -\nabla P$$

(ion energy)

$$\rho \frac{d\varepsilon_i}{dt} = -P_i \nabla \cdot \mathbf{u} - \nabla \cdot \mathbf{q}_i + Q_{ei}$$

(electron energy)

$$\rho \frac{d\varepsilon_e}{dt} = -P_e \nabla \cdot \mathbf{u} - \nabla \cdot \mathbf{q}_e - Q_{ei} + S_L + S_r$$

(radiation transport)

$$\frac{1}{c} \frac{\partial I^\nu}{\partial t} + \Omega \cdot \nabla I^\nu = \eta^\nu - \chi^\nu I^\nu + S^\nu$$

(laser ray-trace and laser absorption)

$$\mathbf{v}_g \cdot \nabla \mathbf{I}_L^k = -\mathbf{v}_{\text{abs}} \mathbf{I}_L^k \quad S_L = \sum_k \mathbf{v}_{\text{abs}} \mathbf{I}_L^k / \mathbf{v}_g^k$$

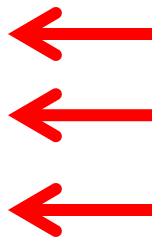
PINOCO (2-D radiation hydrodynamic code)

- two temperature,
 - hydrodynamics
 - ALE-CIP (CIP, RCIP, CCUP)
- thermal transport (electron, ion)
 - flux limited type Spitzer-Harm
 - implicit (9 points-ILUBCG)
- Radiation transport
 - multi-group diffusion type (16~64 groups)
 - implicit (9 points-ILUBCG)
 - Opacity, Emissivity
 - local thermodynamic equilibrium (LTE)
 - non-LTE, collisional radiative equilibrium (CRE)
- Equation of state
 - electron ; Tomas-Fermi model
 - ion ; Cowan model
- Laser ray-tracing
 - 1-D ray tracing
 - inverse-Bremsstrahlung absorption

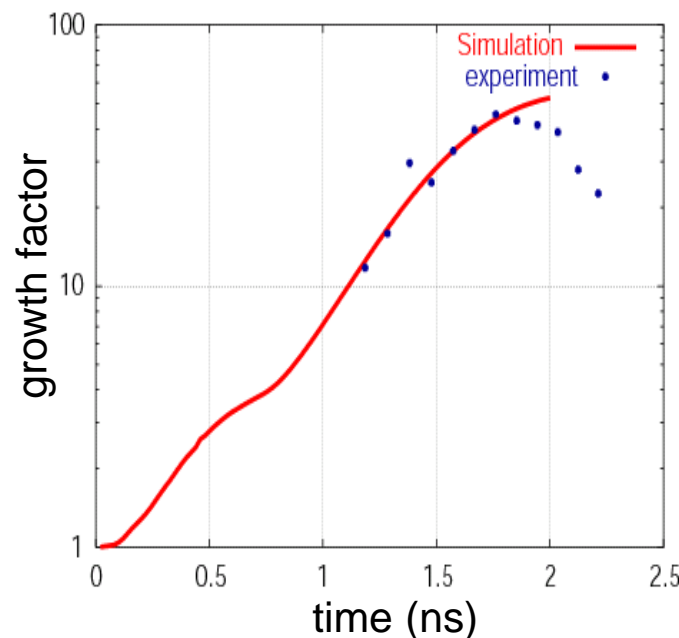
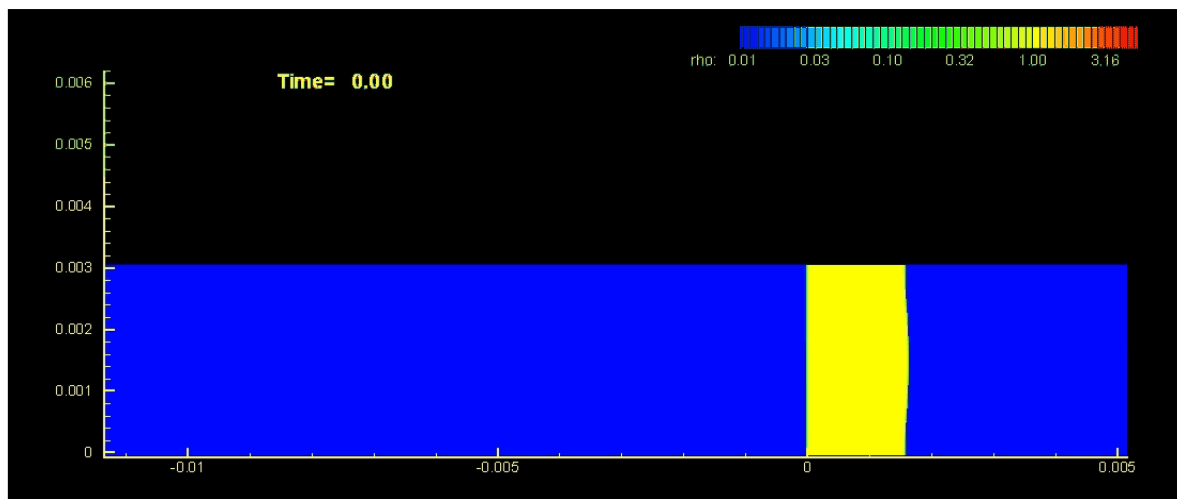
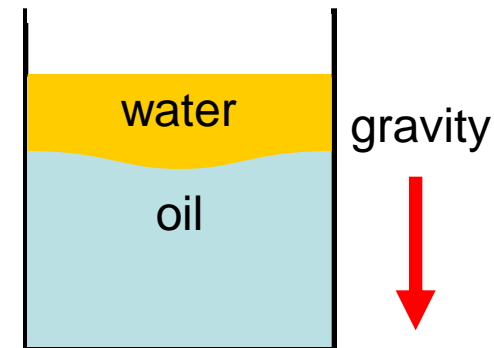
Ablative R-T instability is one of the most critical issue for IFE

Target Laser

CH
 Thickness 16 μm
 Perturbation wavelength 30 μm
 amplitude 0.5 μm



Intensity $7.0 \times 10^{13} \text{ W/cm}^2$
 Wave length 0.53mm



$$\gamma = \sqrt{\frac{kg}{1+kL}} - \beta kv_a$$

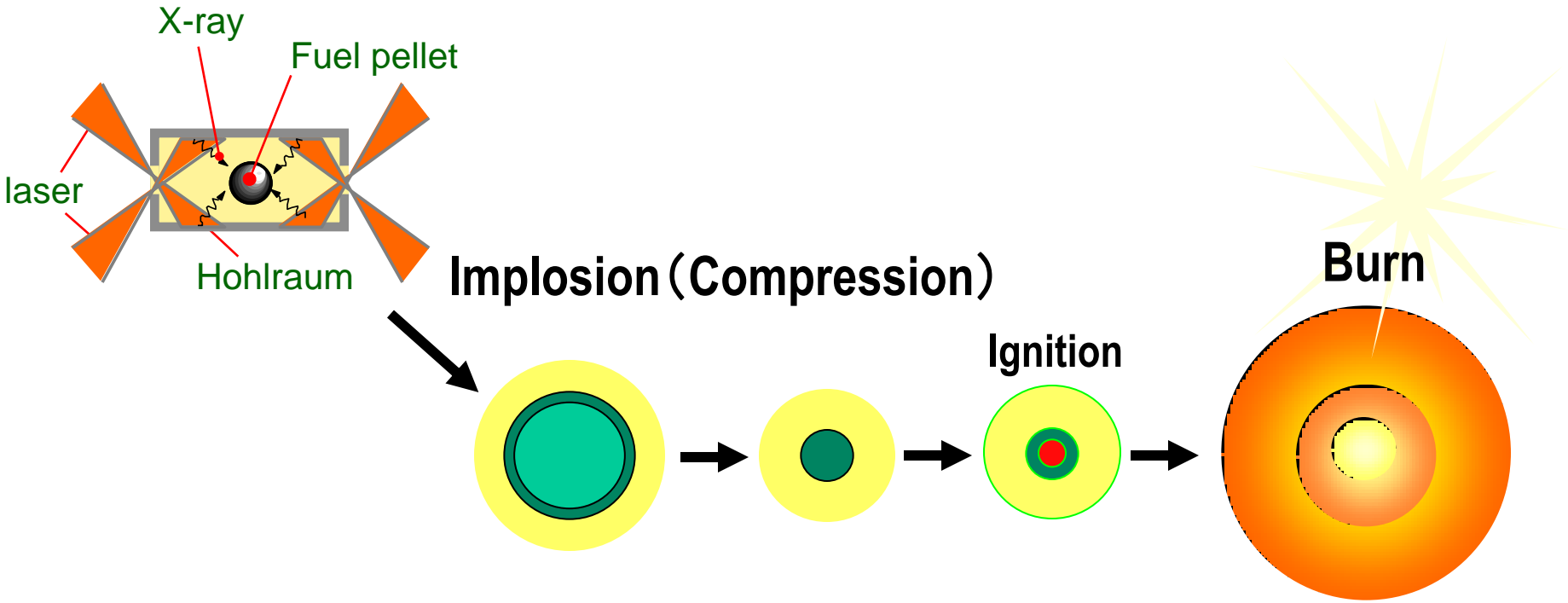
growth rate :

(Takabe, et al, Phys. Fluids, 26,2299 (1983))

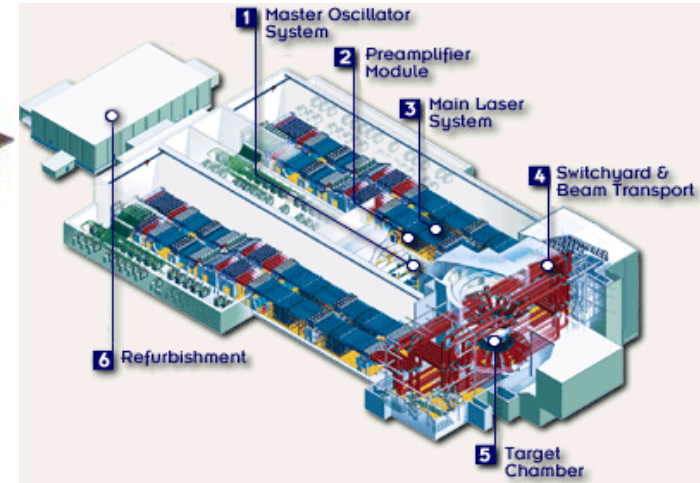
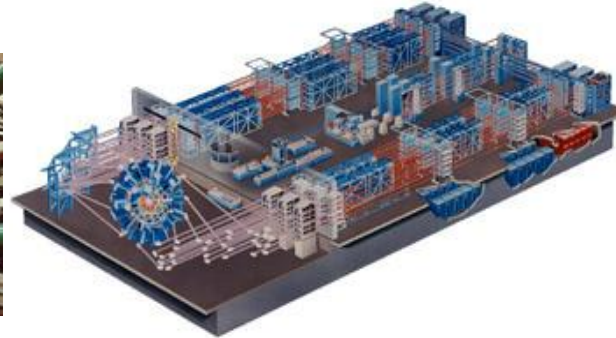
Laser Fusion (Central Ignition, Indirect Drive)

Lawson condition;
1. High temperature (Ion temp.) $T > 10^8\text{K}$
2. High density ($\rho > 200\text{g/cc}$)
3. High arial density($\rho R > 0.3 \text{ g/cm}^2$)

Indirect drive (NIF, LMJ)



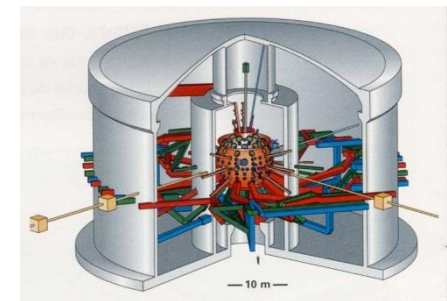
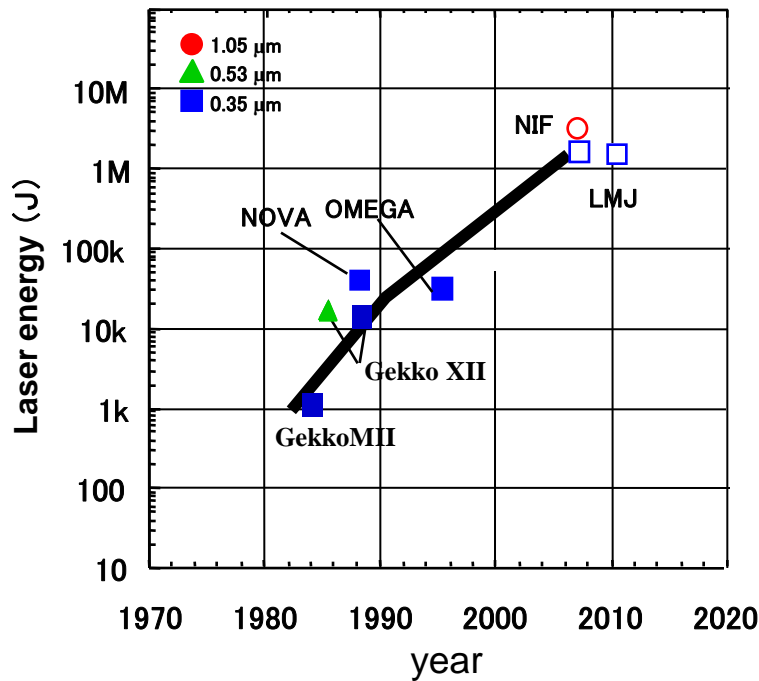
To suppress the R-T instability and other instabilities to achieve ignition, large laser facilities are required.



Gekko XII (Osaka, Japan, 1983)
 25kJ, 0.53 μm , 12 beams
 600 times of solid density

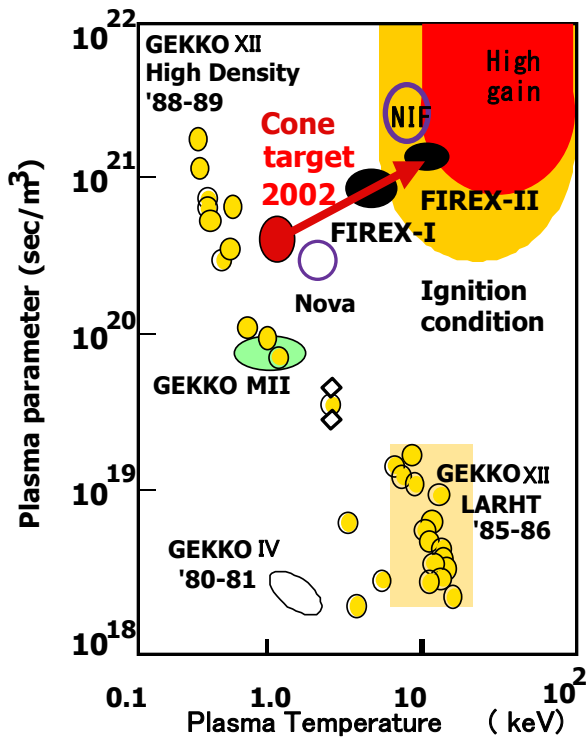
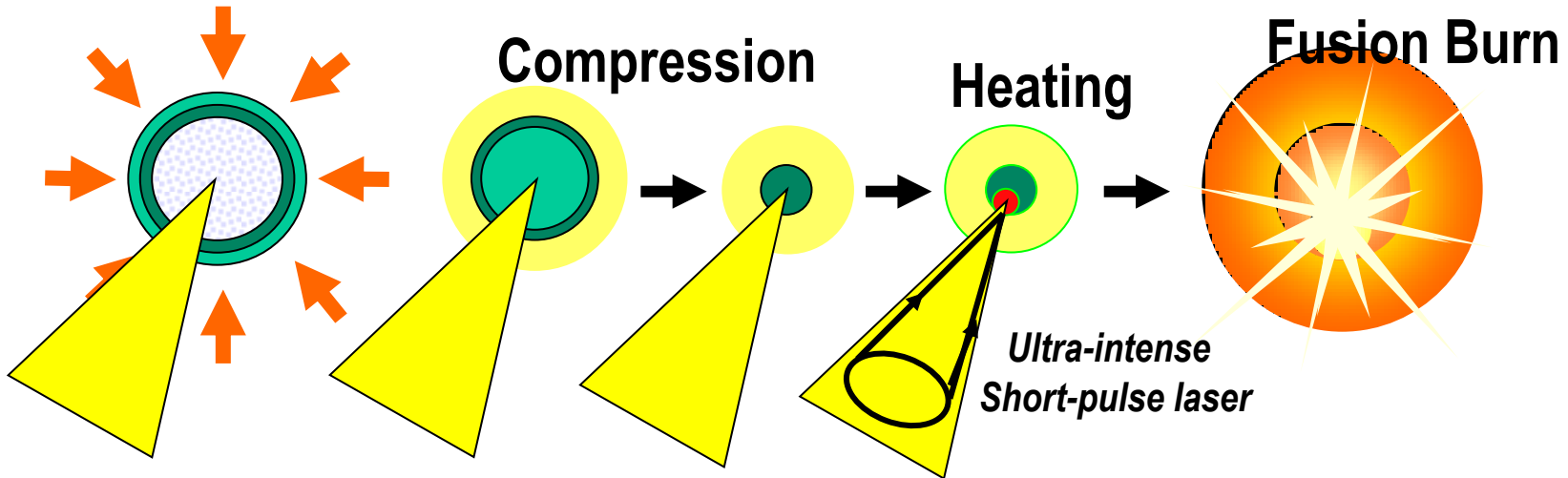
OMEGA (Rochester, USA)
 30kJ, 0.35 μm , 60 beams
 Gain of 0.01

NIF; National Ignition Facility (LLNL, USA)
 1.8MJ, 0.35 μm , 192 beams



LMJ (Bordeaux, France)
 1.8MJ, 0.35 μm , 192 beams

FIREX-I and GXII



- In original FIREX-I design, implosion laser energy (GXII) is larger than the current condition.

FIREX-I

Implosion (GXII) ; 2.5~4.5 kJ in 2ω
(Gaussian)

Heating (LFEX) ; 10 kJ in ω



Peta watt
Laser



1kJ/1ps
(1999–2006)

GXII



10kJ/1ns (1983–)



Target chamber I
(1983–)

LFEX Laser



2008- 10kJ/10ps



Target chamber II
HIPER

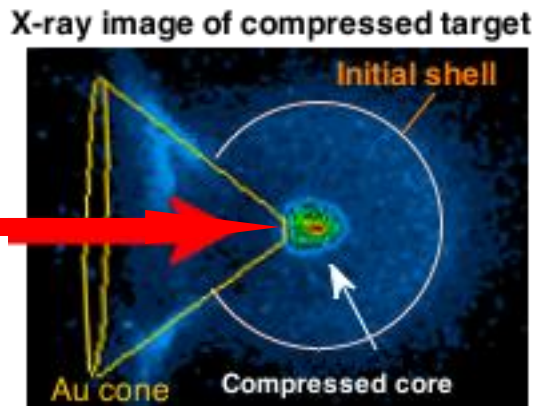
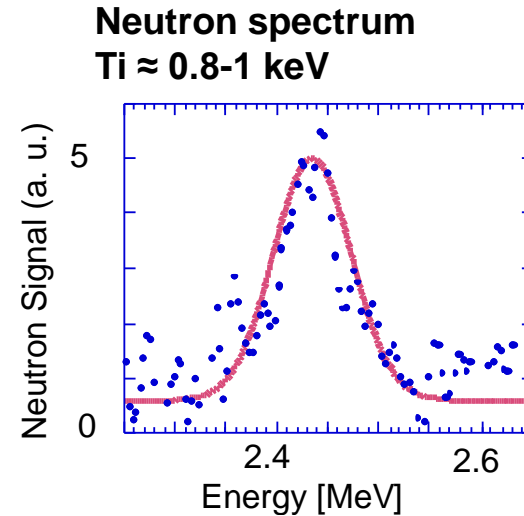
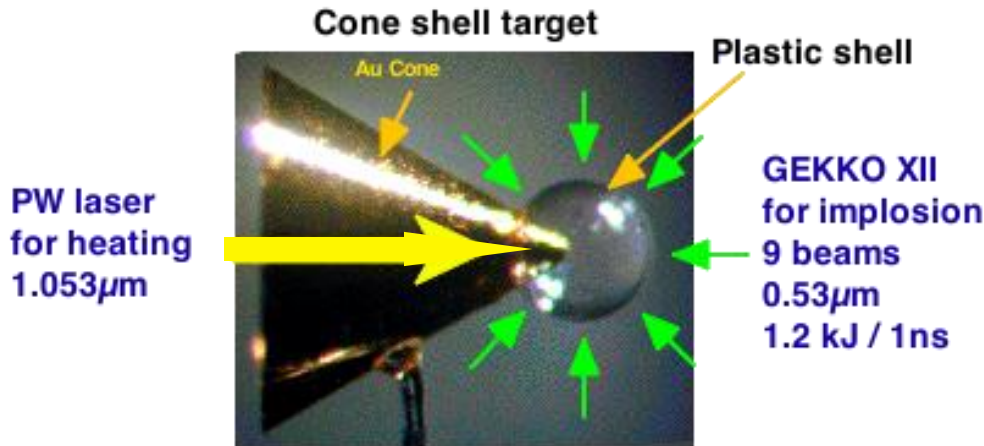
(1998–)

GMII (–2007)

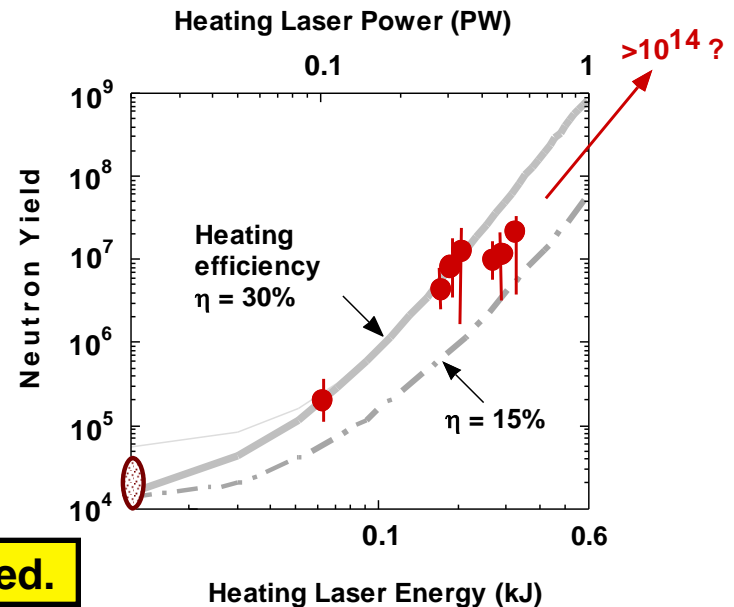




By cone-guiding, heating laser can be introduced very near to the compressed core.



R. Kodama et al. Nature 2002.



Efficient heating with $\eta \sim 0.2$ was demonstrated.

Numerical simulation of cone-guided implosion using 2D radiation-hydro simulation code "PINOCO"

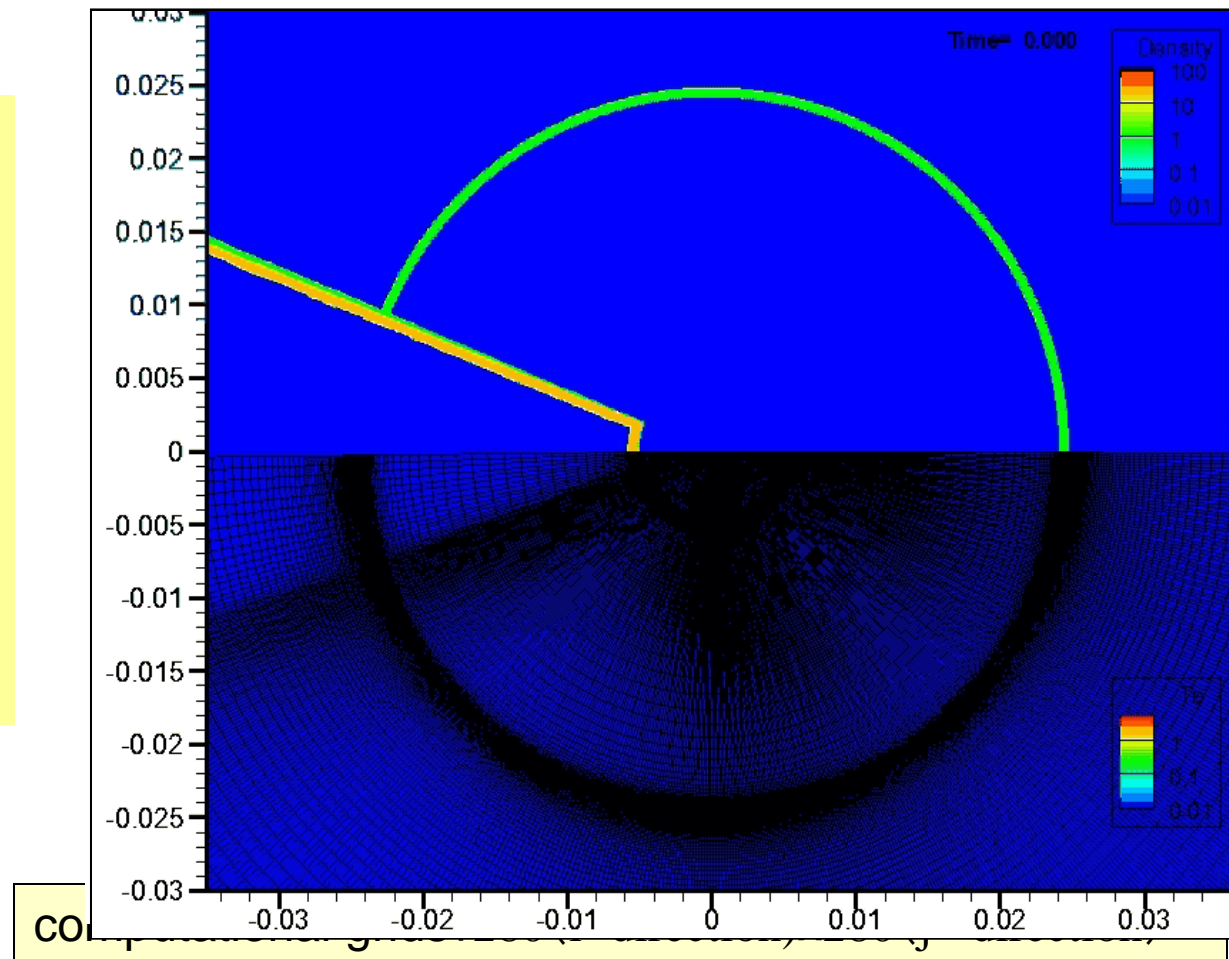
Laser condition

Wavelength : 0.53mm

Energy : 3 kJ (Gaussian, on target, center focus)

PINOCO

- 2 temperature plasma
 - Hydro ALE-CIP method
- Thermal transport
 - flux limited type Spitzer-Harm
 - Implicit (9 point-ILUBCG)
- Laser energy
 - 1-D ray-trace
- EOS
 - Tomas-Fermi
 - Cowan





Example of Radiation hydrodynamic simulation;

Effect of radiation on the gold cone surface inside the shell in cone-guided shell implosion

2-D implosion simulation of CH-D₂ shell with Au cone

Laser condition

Wavelength : 0.53mm

Energy : 3.5 kJ ((for 4π) Gaussian, FWHM=1.5ns)

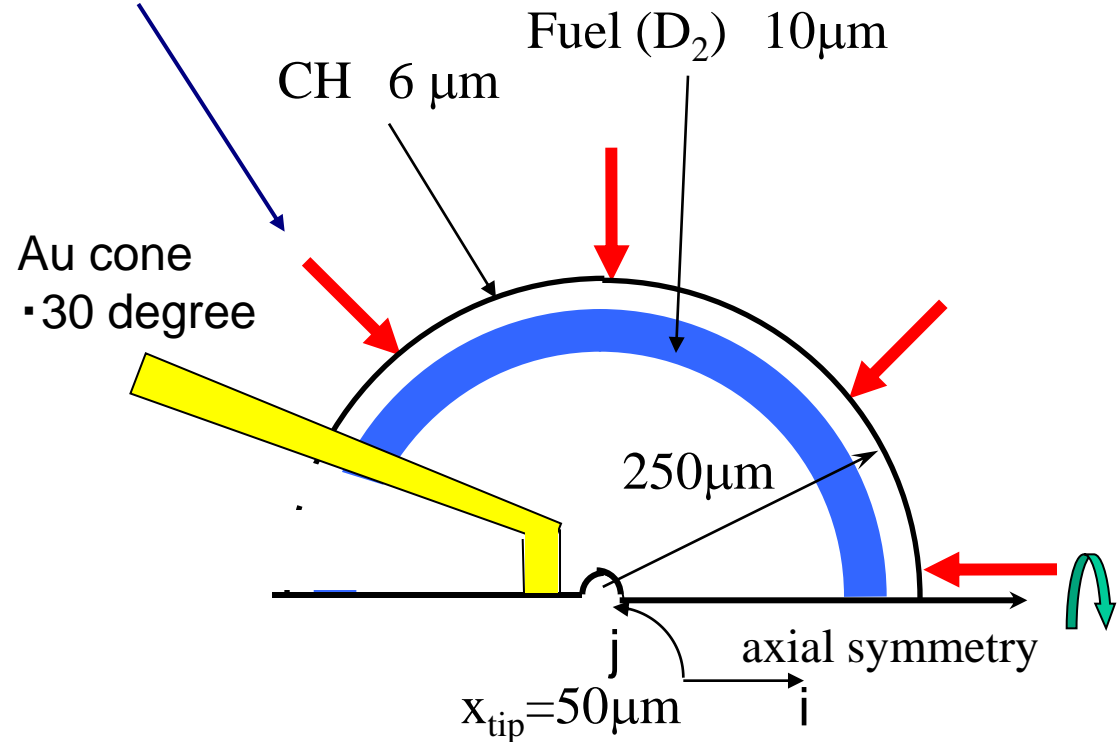
Ray-trace : 1 - D (radial direction)

Radiation transport

Multi-group :

32 group and 16 group

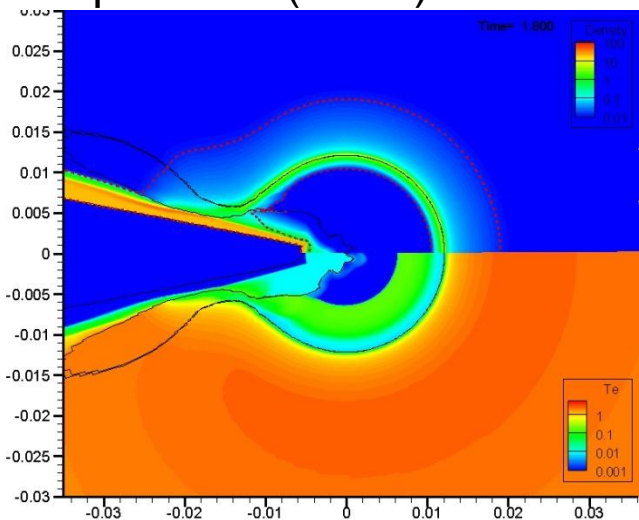
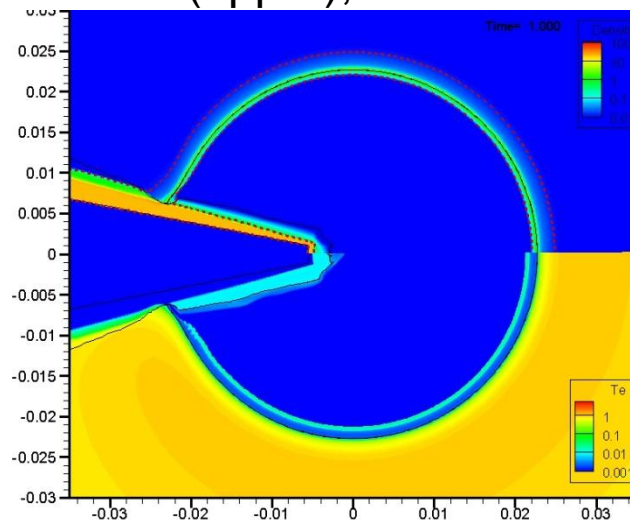
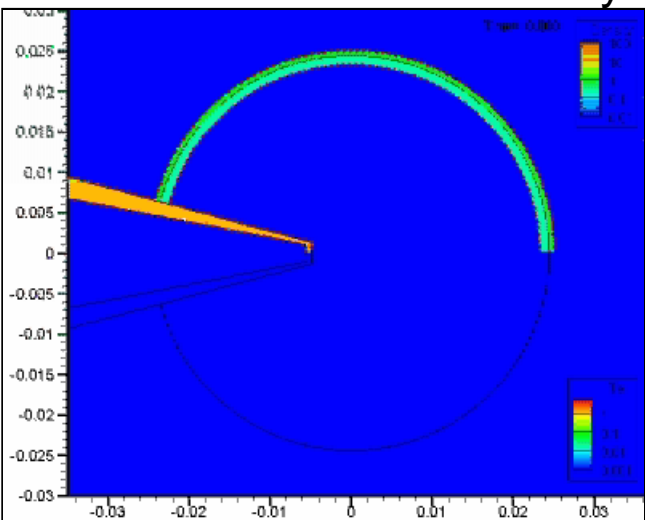
(Two simulations to obtain numerical convergence in photon energy grid.)



computational grids : 300 (i- direction) x 300 (j - direction)

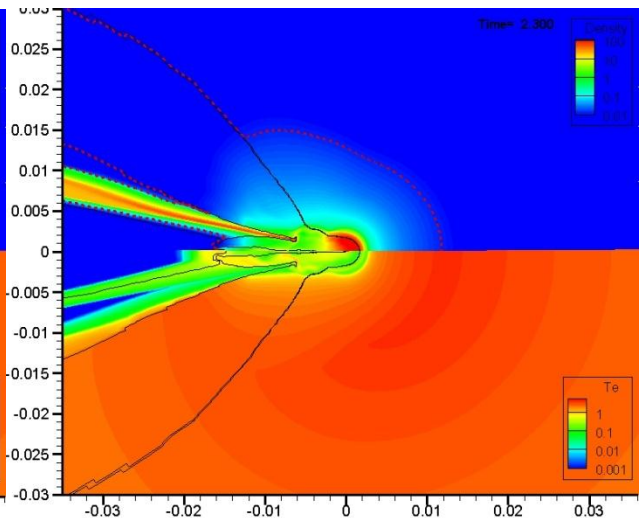
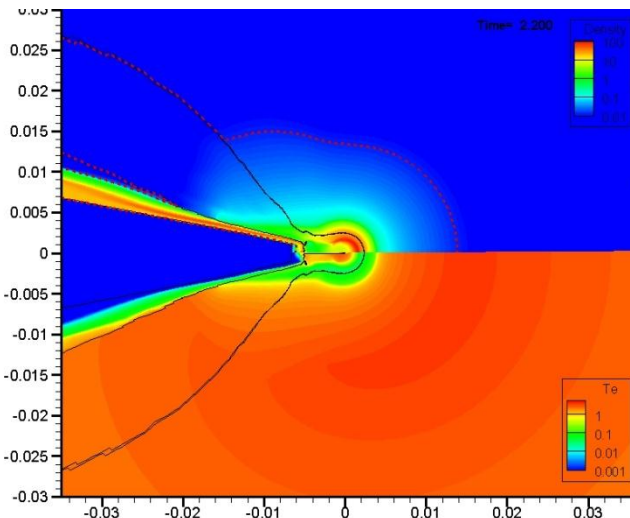
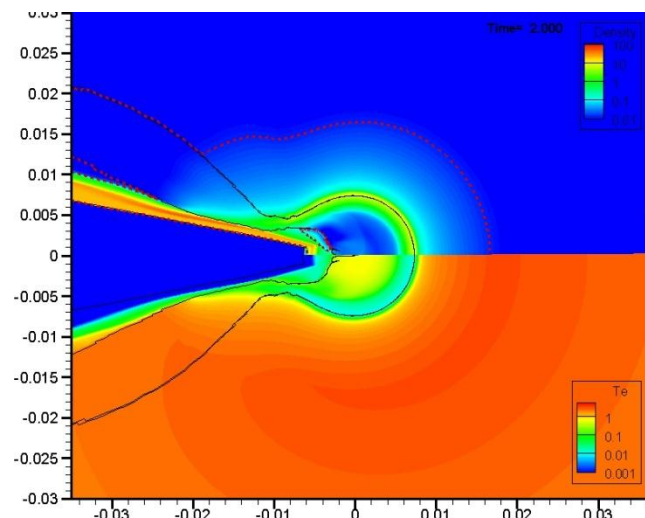
CH-D₂ shell + Au cone w/o coating

mass density contours (upper), and electron temperature (lower)



1.00 ns

1.80 ns



2.00 ns

2.20 ns

2.30 ns

2-D implosion simulation of a CH-D₂ shell with Au coated cone

Laser condition

Wavelength : 0.53mm

Energy : 3.5 kJ ((for 4π) Gaussian, FWHM=1.5ns)

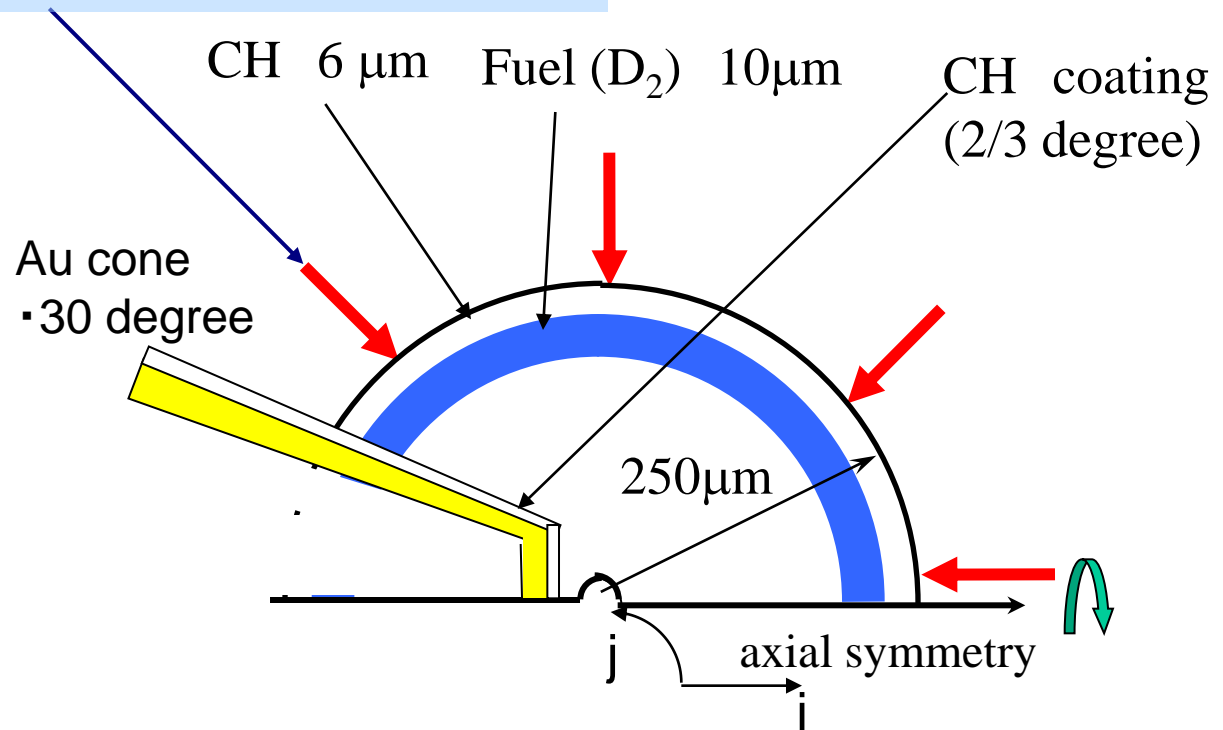
Ray-trace : 1 - D (radial direction)

Radiation transport

Multi-group :

32 group and 16 group

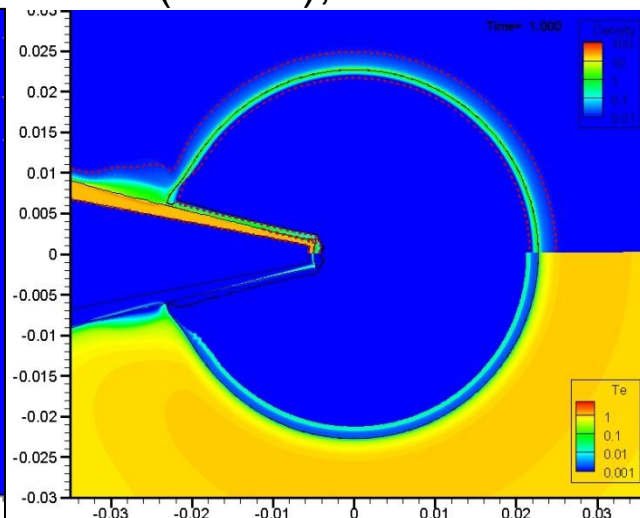
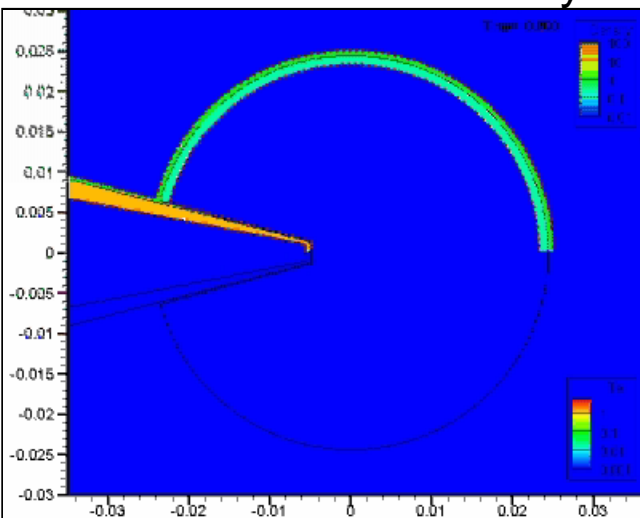
(Two simulations to obtain numerical convergence in photon energy grid.)



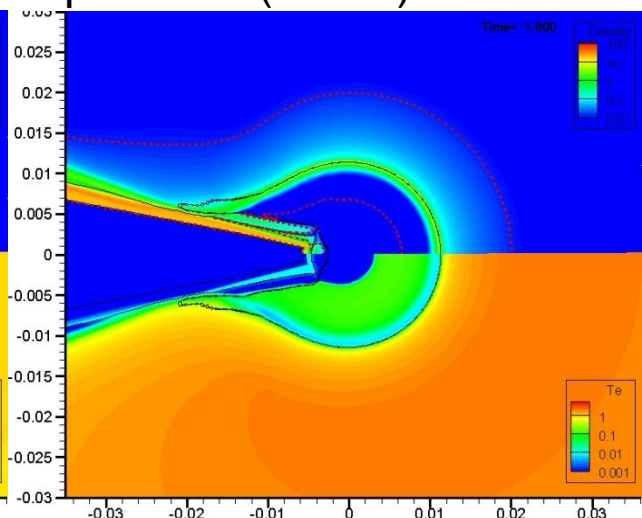
computational grids : 300 (i- direction)x300 (j - direction)

CH-D₂ shell + gold cone with CH coating

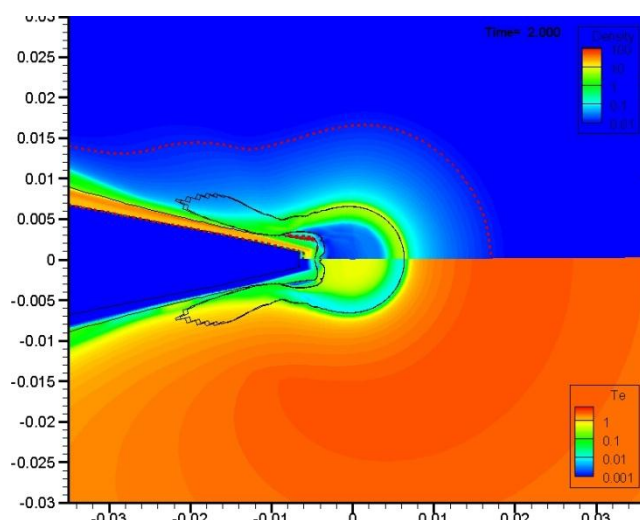
mass density contours (above), and electron temperature (below)



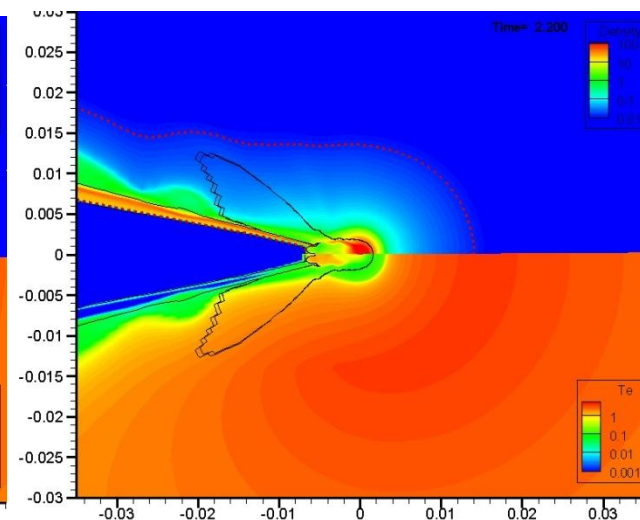
1.00 ns



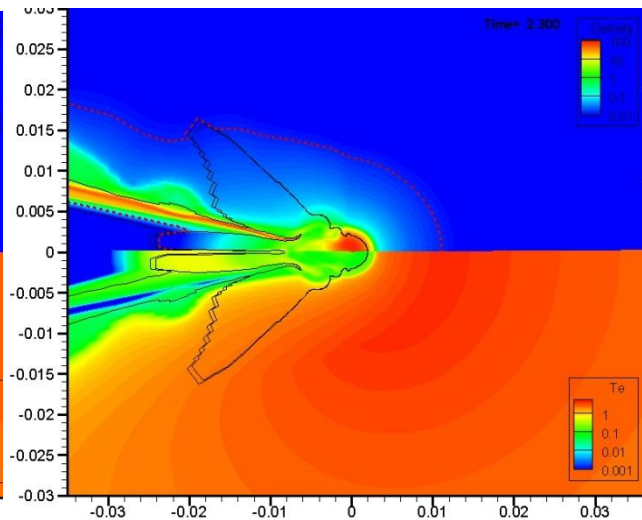
1.80 ns



2.00 ns



2.20 ns



2.30 ns

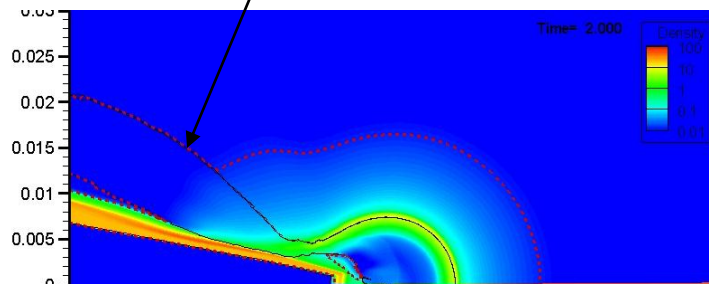


D_2 fuel is leaking near Au surface in both cases. But it is less amount of leaking D_2 in case of with coating. This fact was not observed in a simulation without radiation transport.

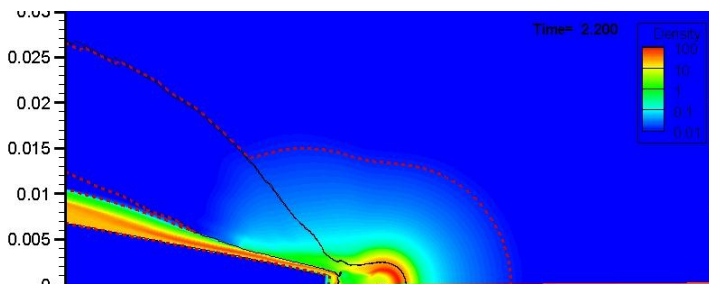
Step at shell-cone interface may reduce the leaking. (R. B. Stephens, et al., PoP, **12**, 056312, 2005)

w/o coating

contact surface



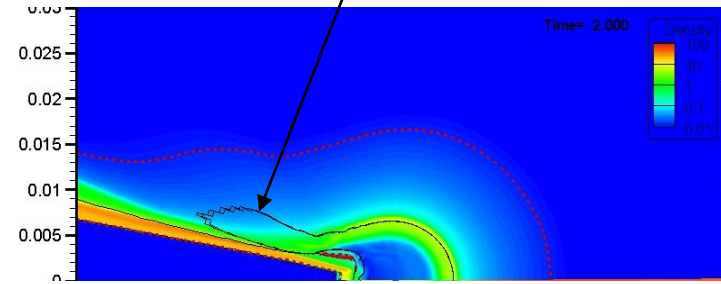
t=2.00 ns



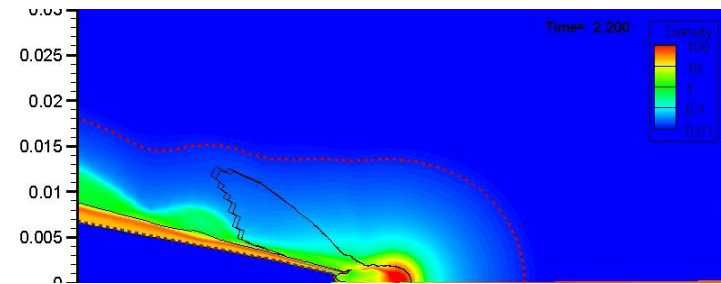
t=2.20 ns

with coating

contact surface



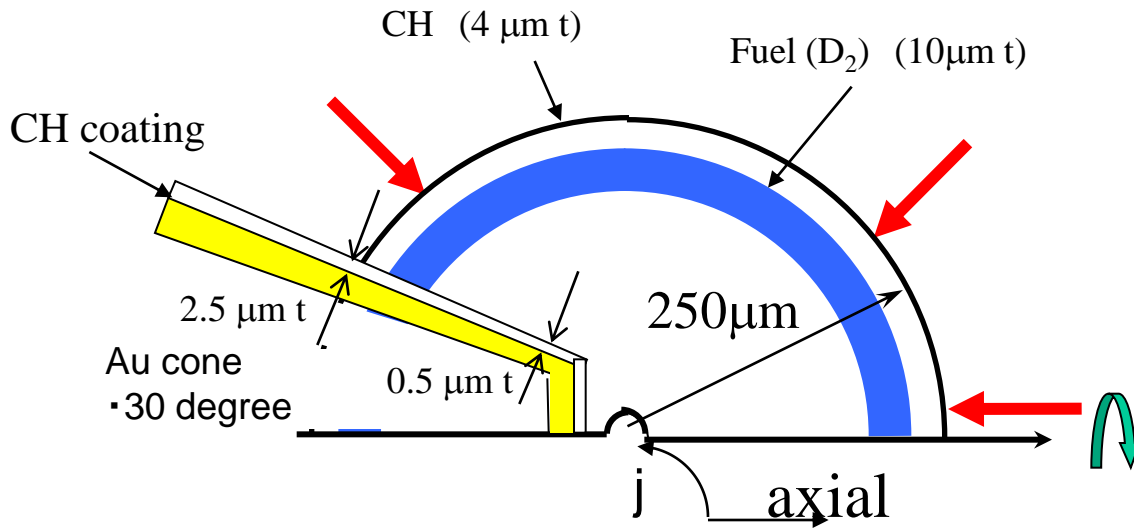
t=2.00 ns



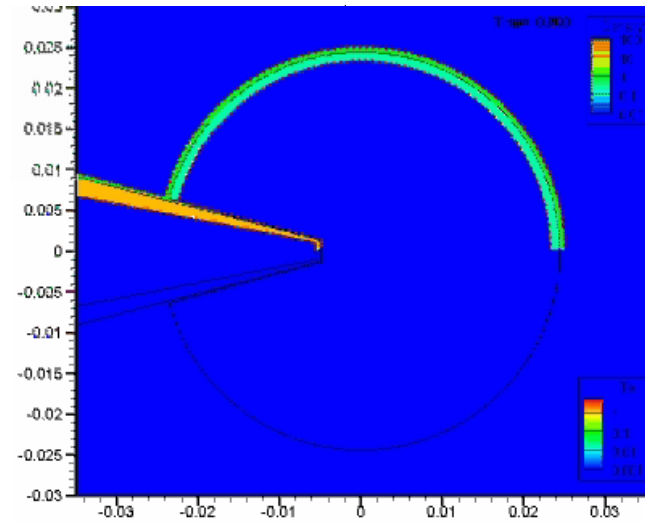
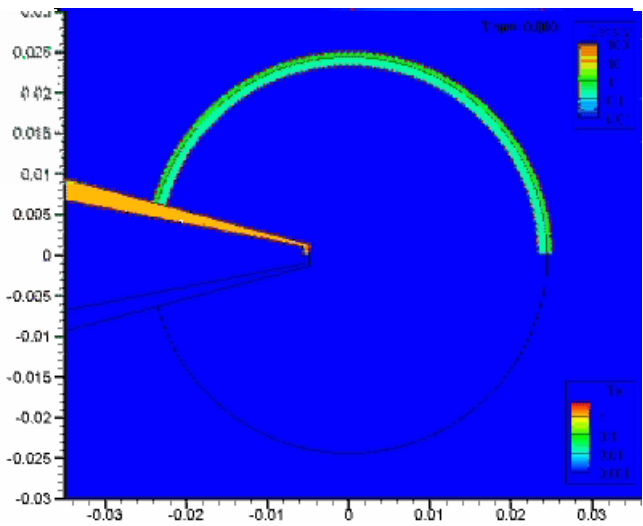
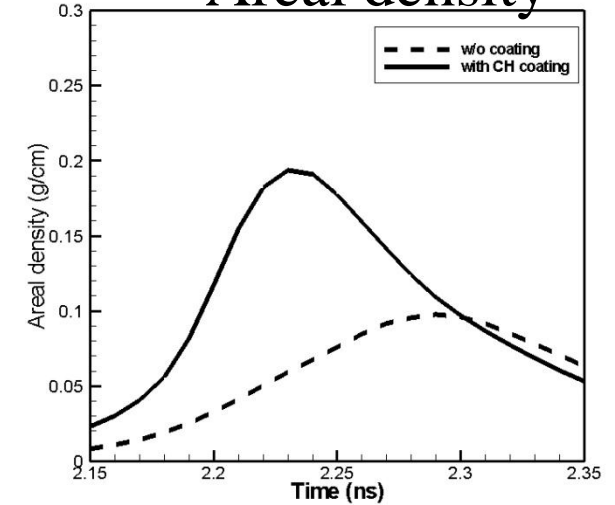
t=2.20 ns



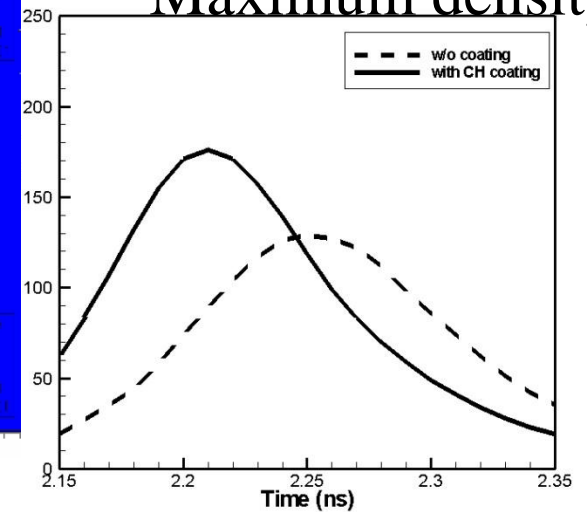
Our previous work suggests that CH coating on the gold cone is effective to tamp ablated gold plasma which is expanding into the shell. (Nagatomo *et al.* PoP 2007)



Areal density



Maximum density





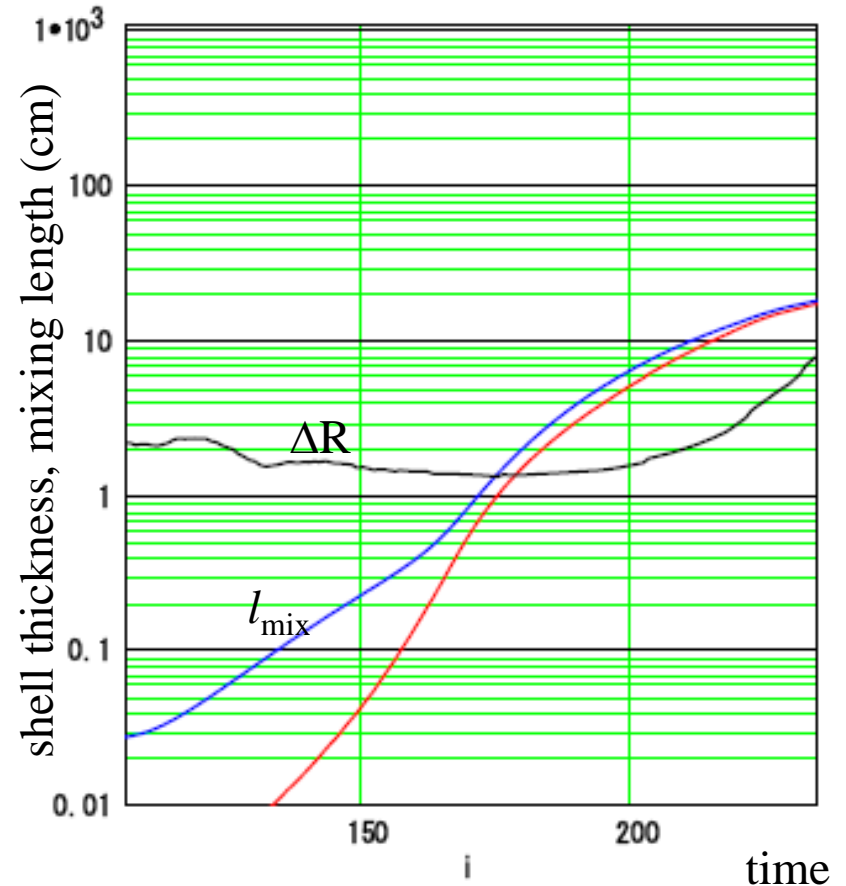
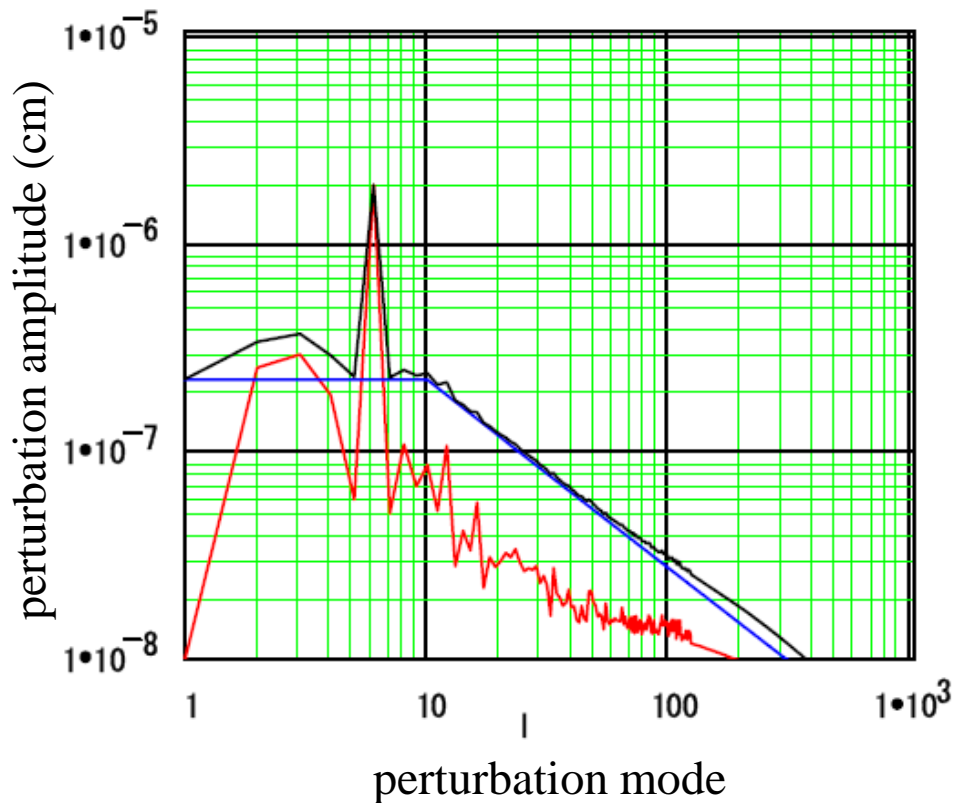
Example of Radiation hydrodynamic simulation;

Effect of Rayleigh-Taylor instability in cone-guided implosion.



1D simulation + perturbation models ; A standard target for GXII may be broken up before maximum compression

- ILESTA1D-FP
- + measured target surface roughness + Nakai-Azechi's imprint model

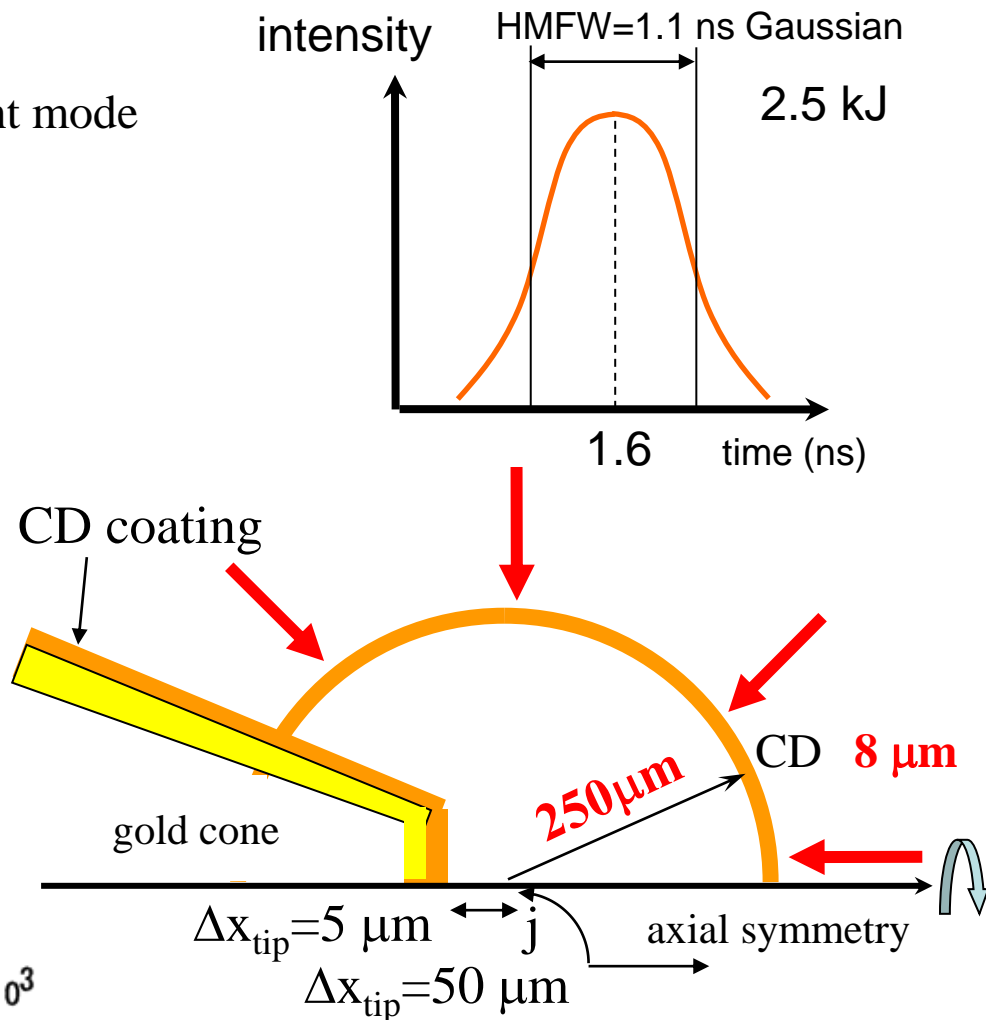
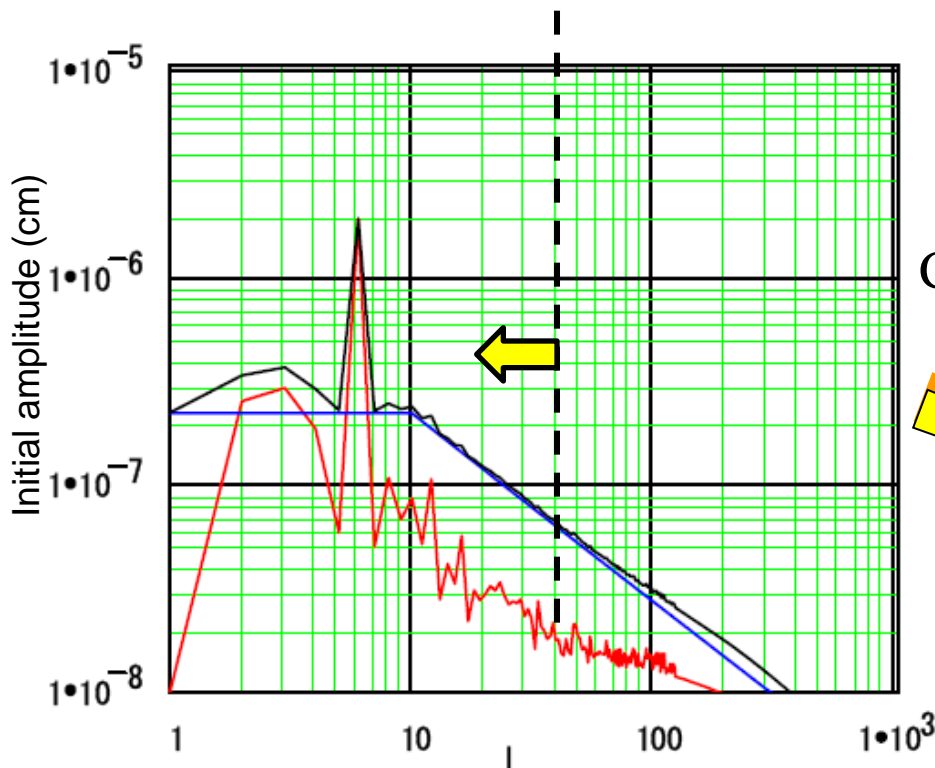




Detail 2-D rad-hydrodynamics simulations for fast ignition are performed to evaluate the implosion performance.

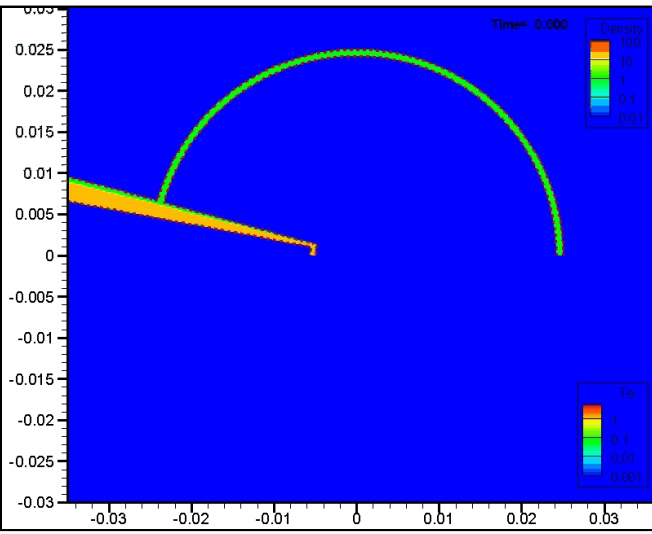
CD shell target ; multi-mode ($l=2\sim 40$) , initial target surface perturbation is given in PINOCO-2D simulation

- target surface roughness measured + Nakai-Azechi initial imprint mode
- Uniform laser irradiation
- computational grid (310x352)

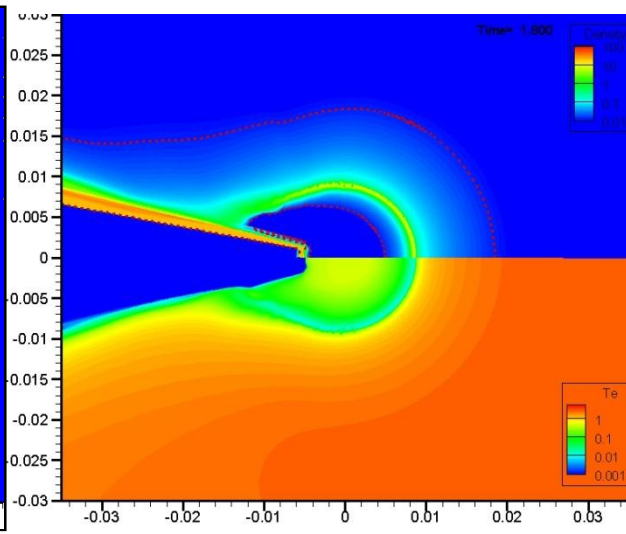




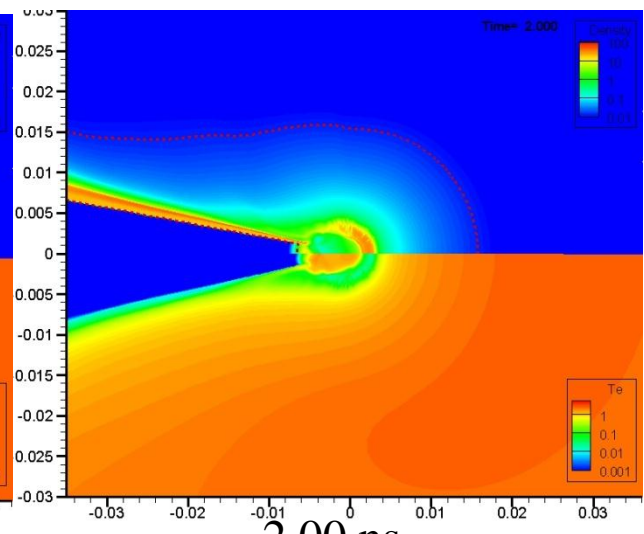
overview of the implosions (cone-guided **without initial perturbation** on the shell target surface) $Y_n=9.7 \times 10^7$



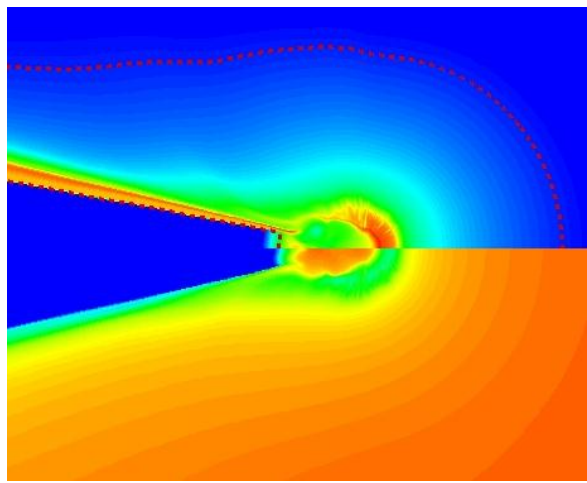
0.00 ns



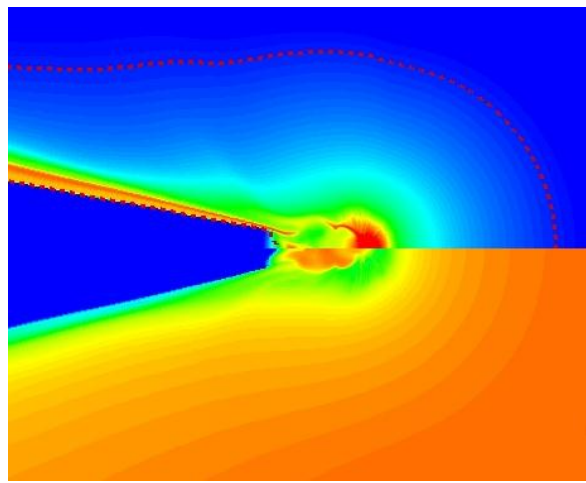
1.80 ns



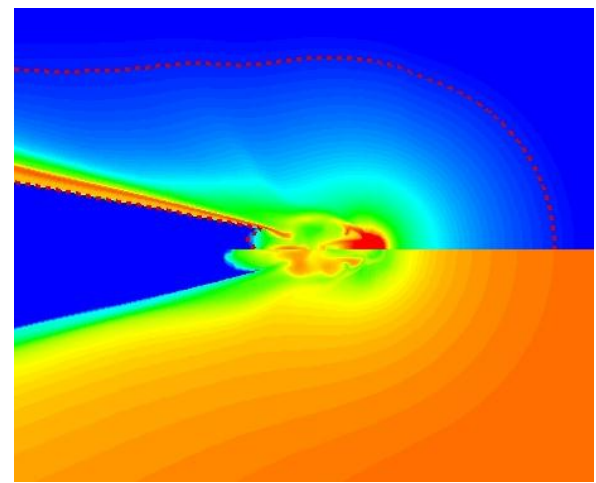
2.00 ns



2.02 ns



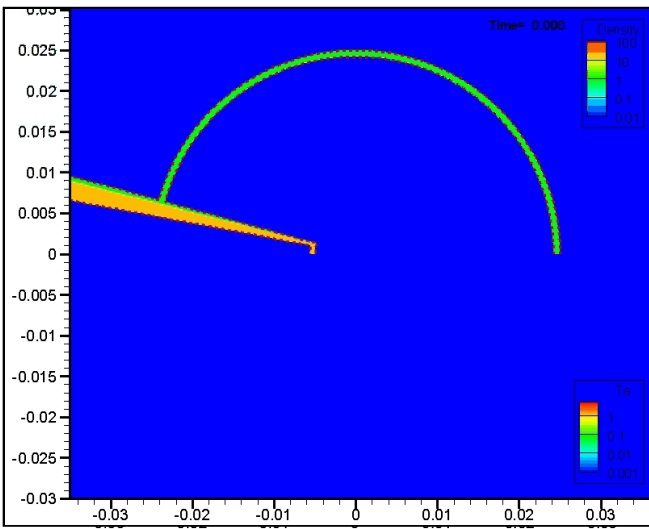
2.06 ns



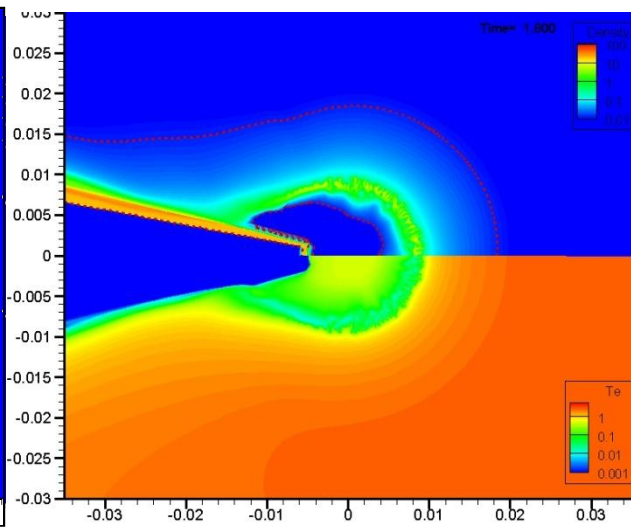
2.10 ns



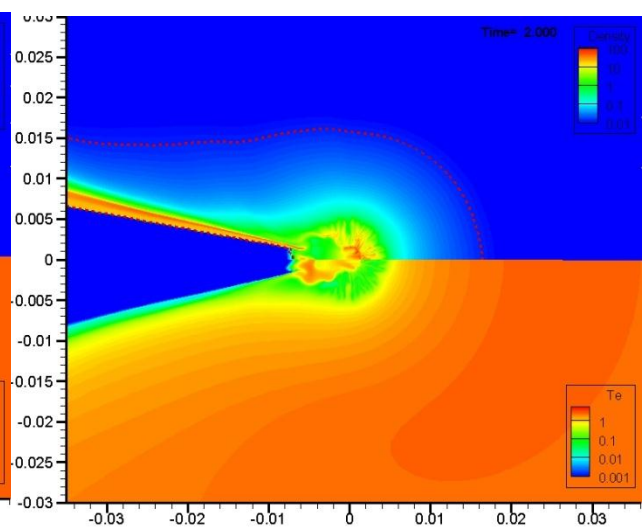
overview of the implosions (cone-guided **with initial perturbation** on the shell target surface) $Y_n=5.9 \times 10^7$



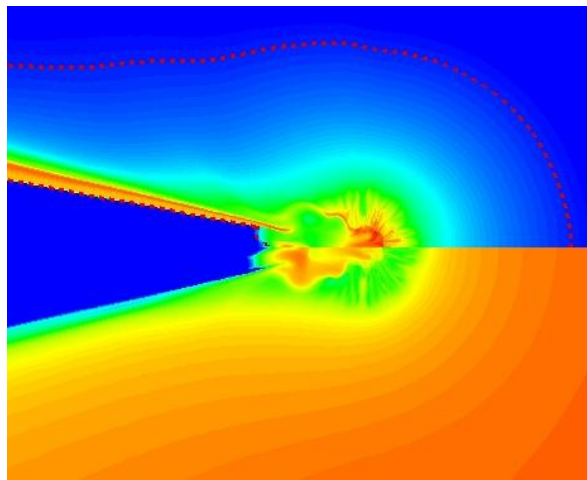
0.00 ns



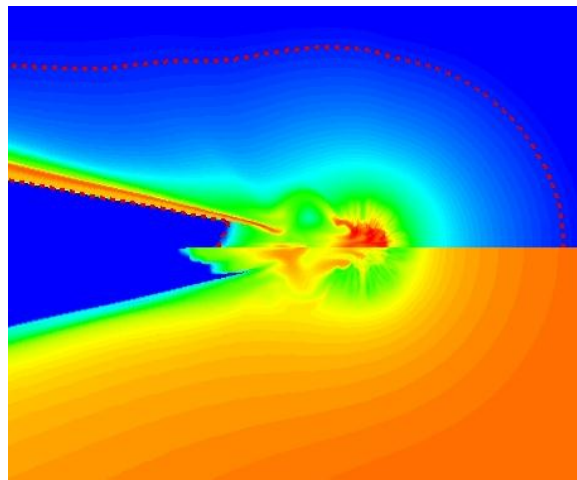
1.80 ns



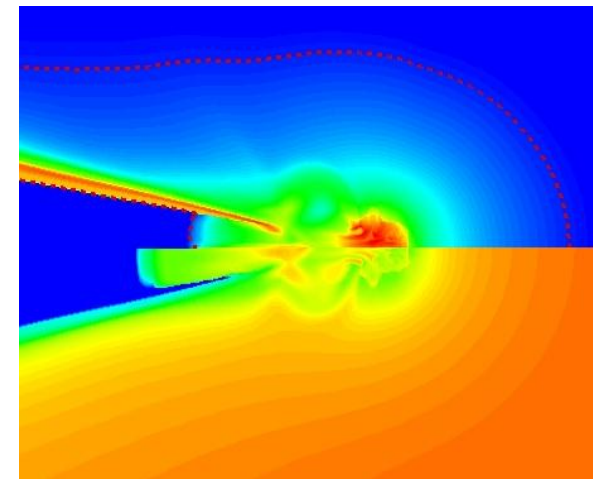
2.00 ns



2.02 ns



2.06 ns

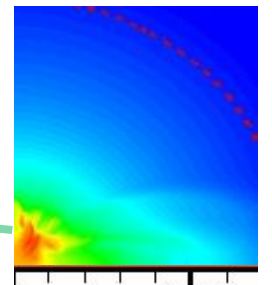
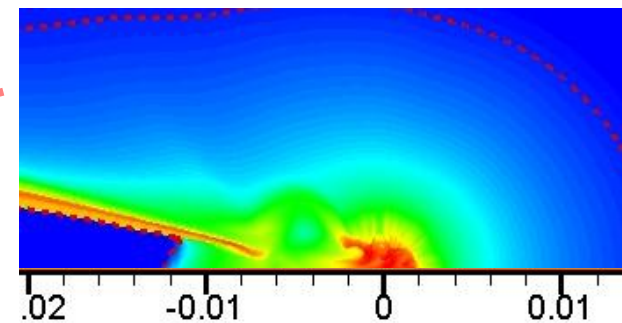
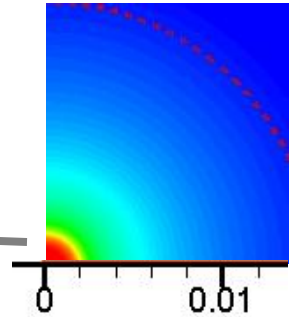
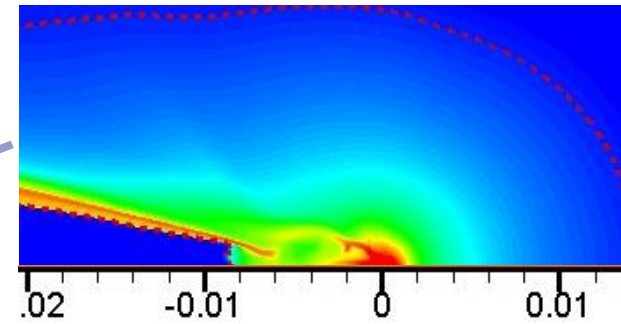
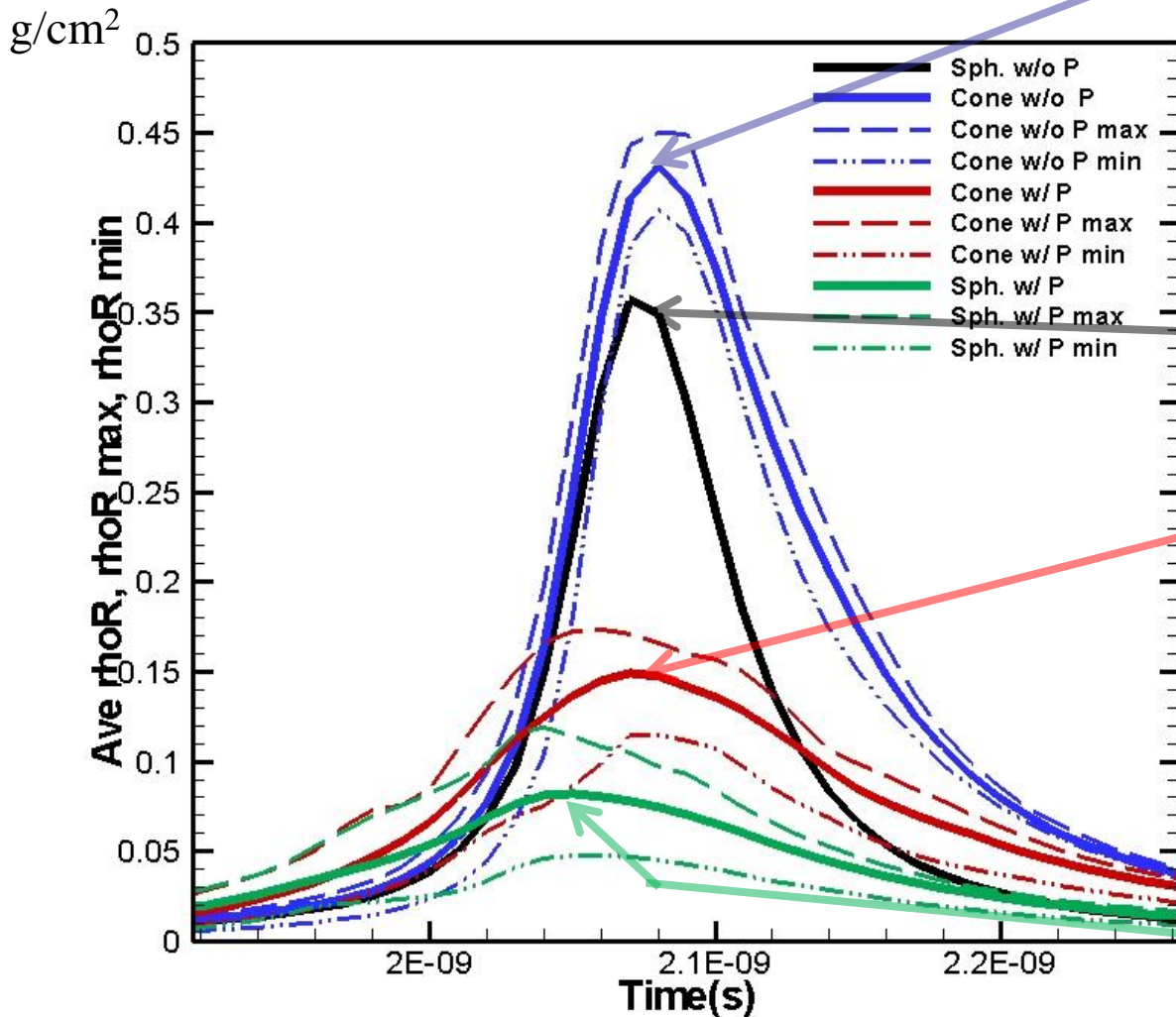


2.10 ns



ρR is reduced by the hydrodynamics instability

Angular dependence of ρR (average, minimum, maximum) are compared
(only quadrant spheres parts are evaluated)



Cone tip is destroyed before the maximum compression time, if Rayleigh-Taylor instability is strongly grown.



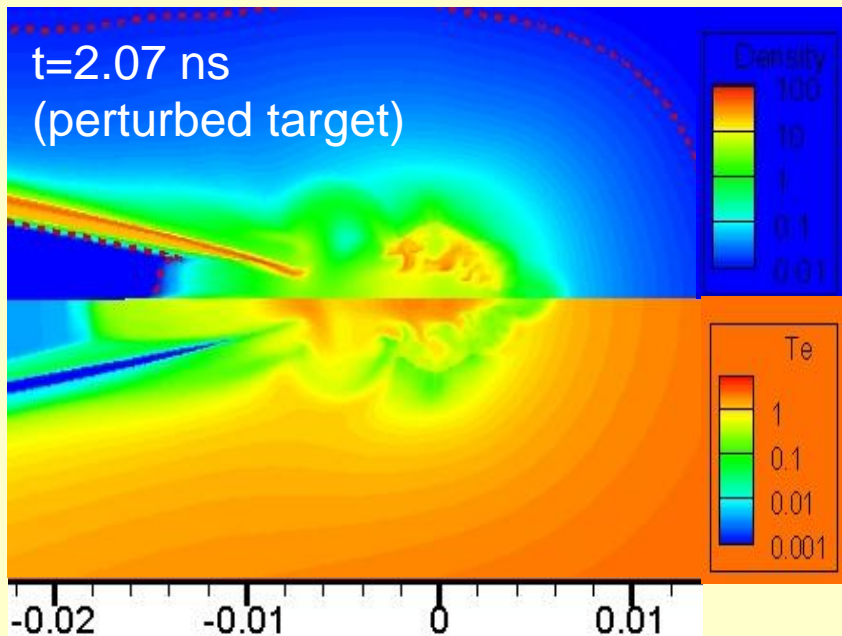
During the imploding phase, due to the hydrodynamic instability which is occurred in acceleration phase, plasma expands into the shell, and the pressure inside the shell is increased.

Thus, the imploding shell is decelerated, and maximum mass density is dropped. Also, the inner plasma collides with the tip of the cone, and it is destroyed before the maximum compression time.

With initial perturbation

($t=1.97$ ns, 100 ps before maximum compression, tip is broken.)

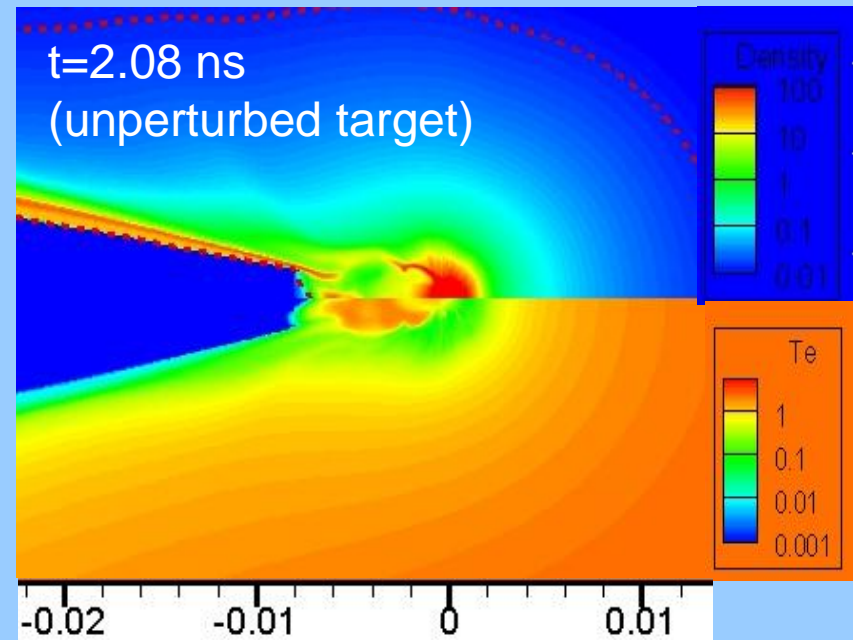
$$Y_n = 5.9 \times 10^7$$



Without initial perturbation

(clean cone-guided implosion)

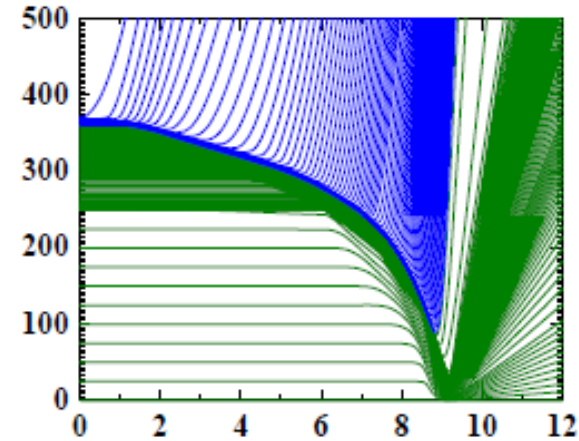
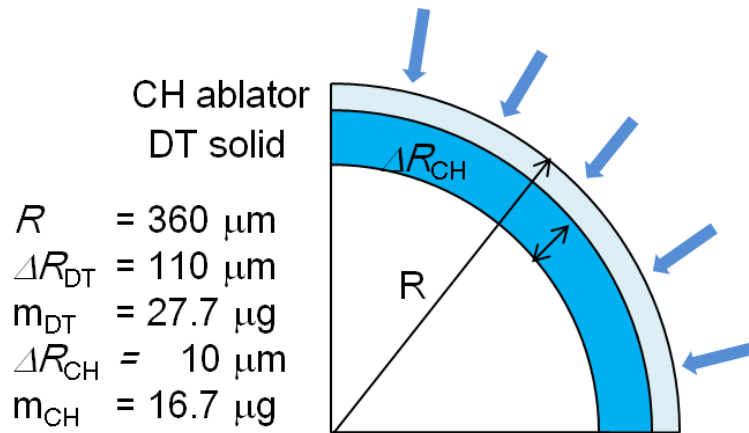
$$Y_n = 9.7 \times 10^7$$



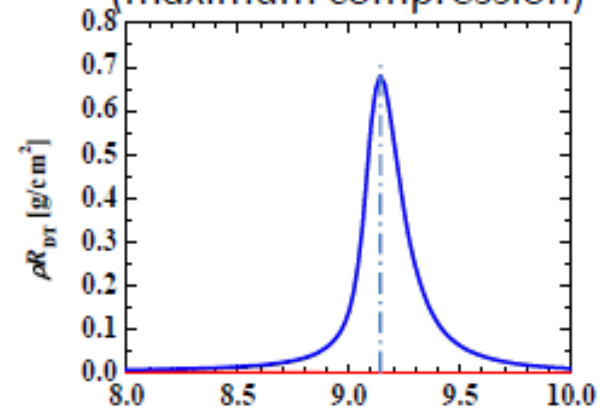
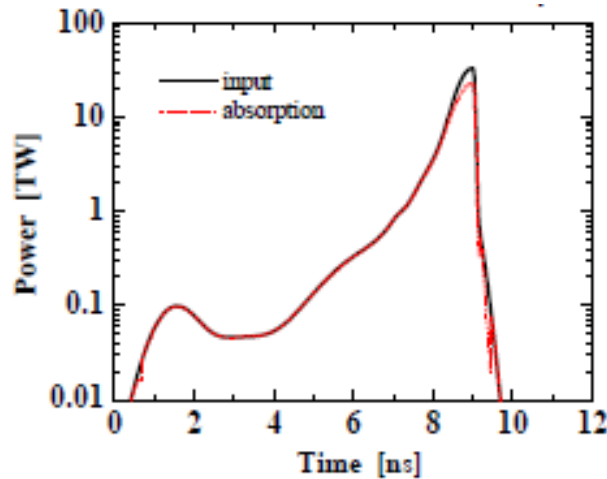
Slow implosion design FIREX-1.5

Target design using 1-D simulation

20kJ, 0.35 μm

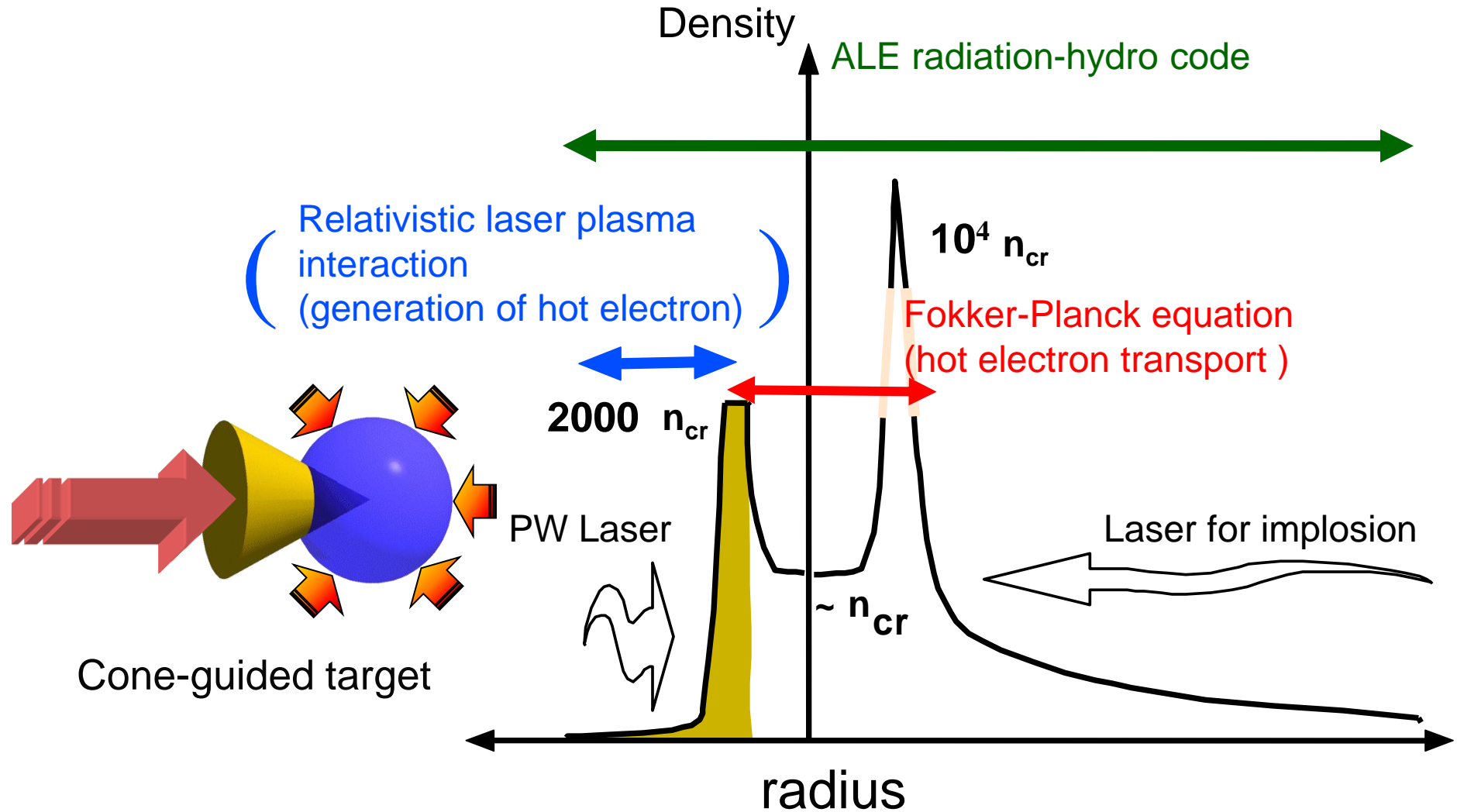


$\rho R_{\text{DT}} = 0.68 \text{g/cm}^2$ at $t = 9.14 \text{ns}$
(maximum compression)



FI³ Project

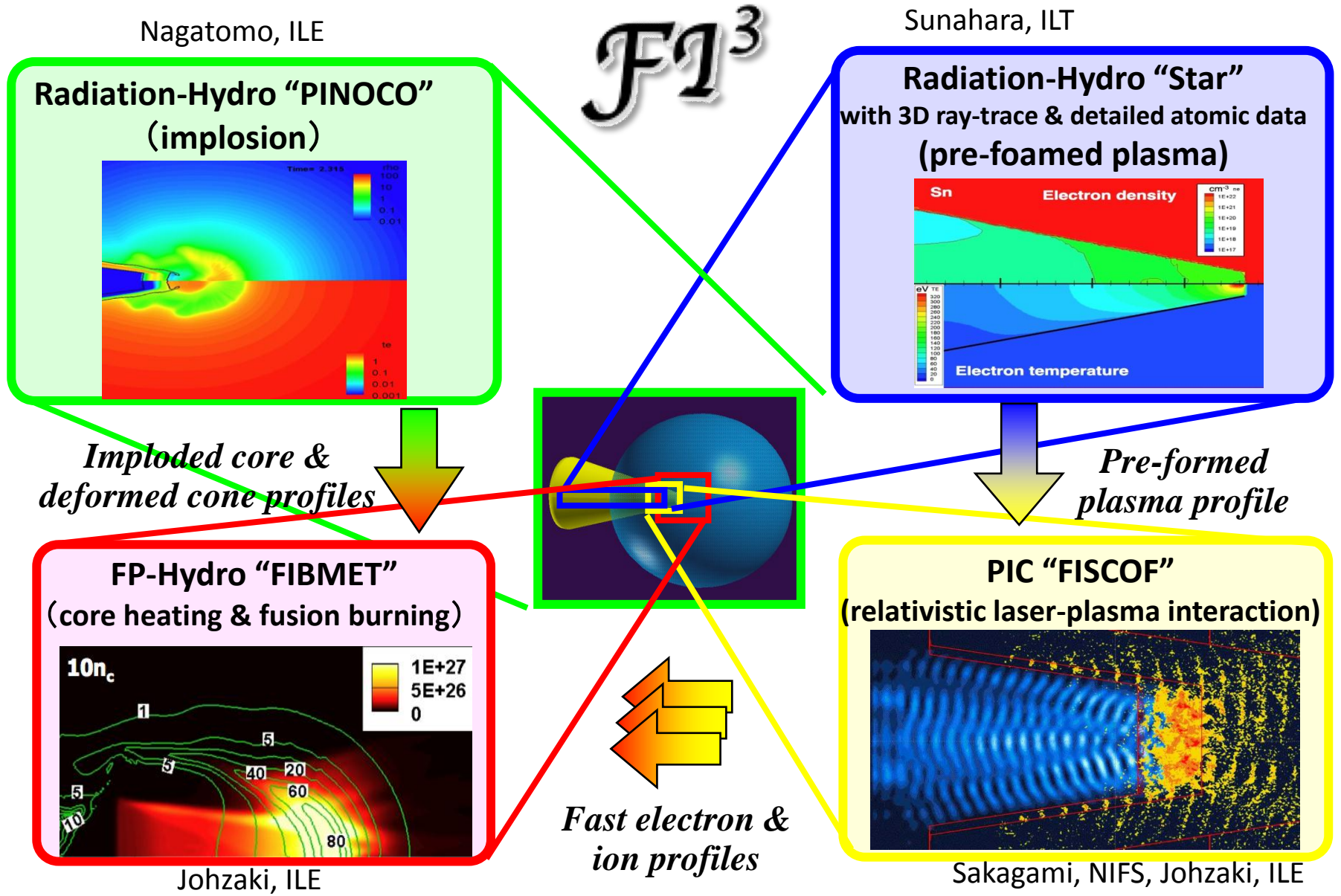
Fast Ignition Integrated Interconnecting code



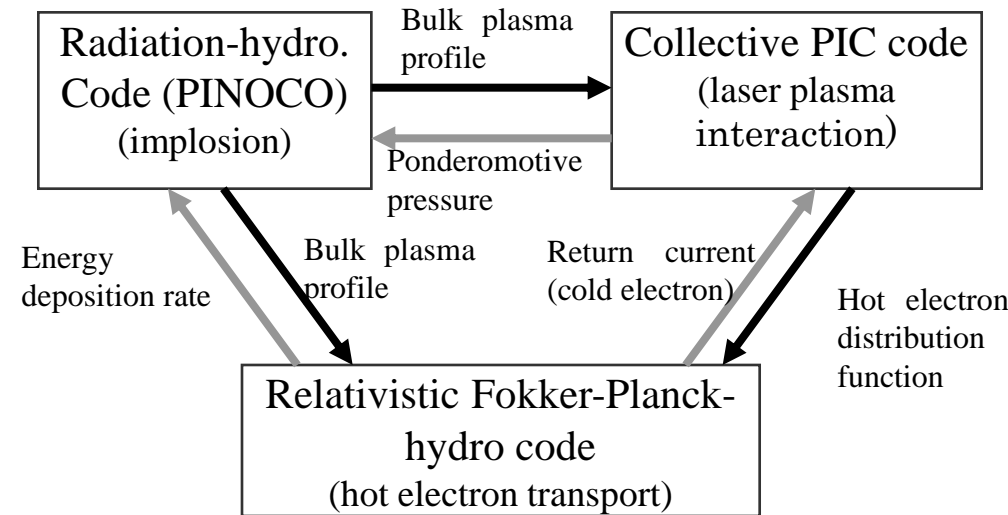
Integrated Simulations



FI^3 (Fast Ignition Integrated Interconnecting) Code System

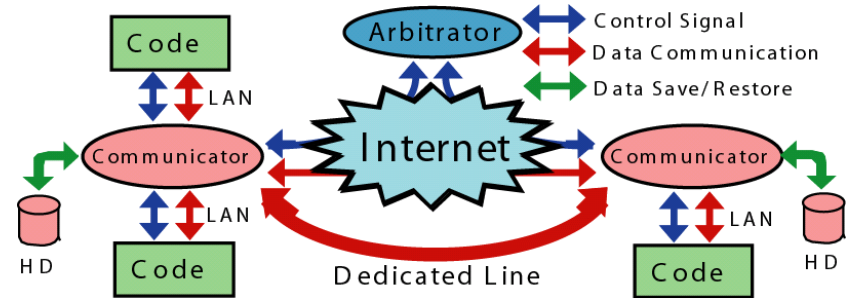


(Distributed Computing Collaboration Protocol)



Data flow in FI³ system. Black arrows are already executable data flows, and gray arrows are next plan to be considered.

- ☀ dynamic negotiation for communication pair
 - Code can be invoked at an arbitrary computer.
- ☀ asynchronous communication
 - Data will be automatically saved/restored.



- ☺ Code (user' s simulation program)
 - does not transfer data directly to another code
- ☺ Communicator
 - receive data from sender code
 - forward data between different sites
 - send data to receiver code
- ☺ Arbitrator
 - manage information of codes
 - control data communication