Vlasov-Fokker-Planck (VFP) Simulation

Kinetic modelling on hydro timescale

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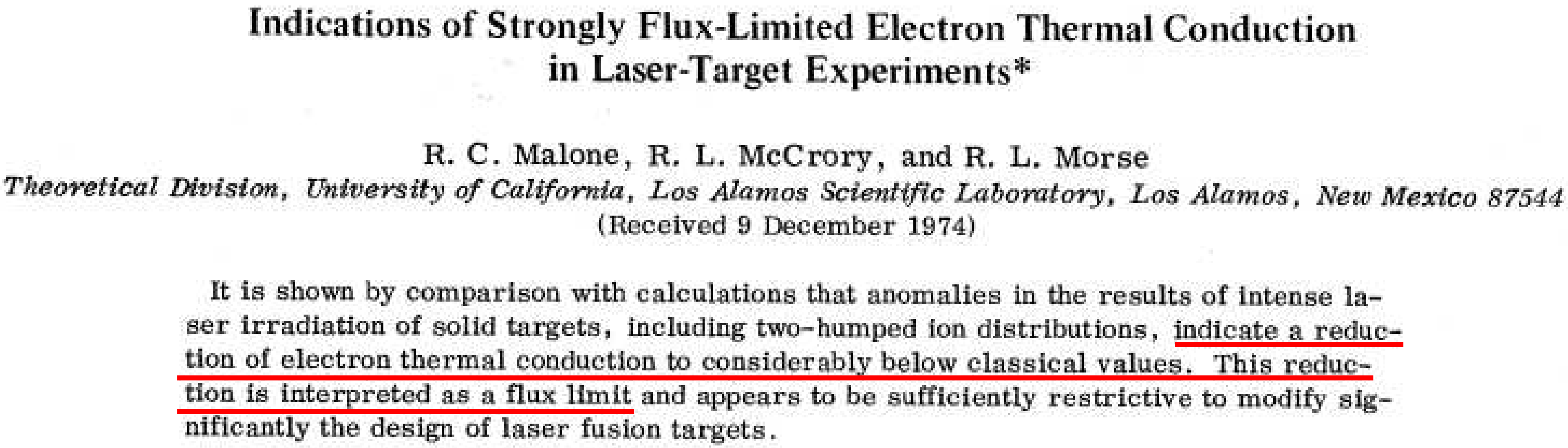
With thanks to many others:
Eduardo Epperlein, Graham Rickard, Richard Town, Jonathan Davies, Robert Kingham,
Alex Robinson, William Hornsby, Mark Sherlock, Chris Ridgers, Michail Tzoufras

especially Michail Tzoufras for recent work

Where it started for laser-plasmas …



Indications of Strongly Flux-Limited Electron Thermal Conduction in Laser-Target Experiments*

R. C. Malone, R. L. McCrory, and R. L. Morse
Theoretical Division, University of California, Los Alamos Scientific Laboratory, Los Alamos, New Mexico 87544
(Received 9 December 1974)

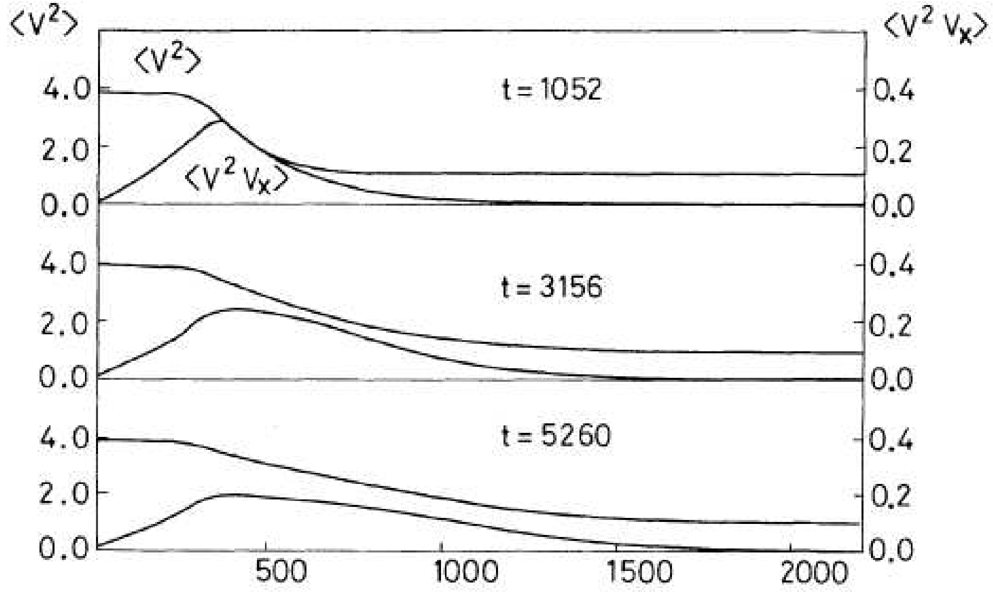
It is shown by comparison with calculations that anomalies in the results of intense laser irradiation of solid targets, including two-humped ion distributions, indicate a reduction of electron thermal conduction to considerably below classical values. This reduction is interpreted as a flux limit and appears to be sufficiently restrictive to modify significantly the design of laser fusion targets.

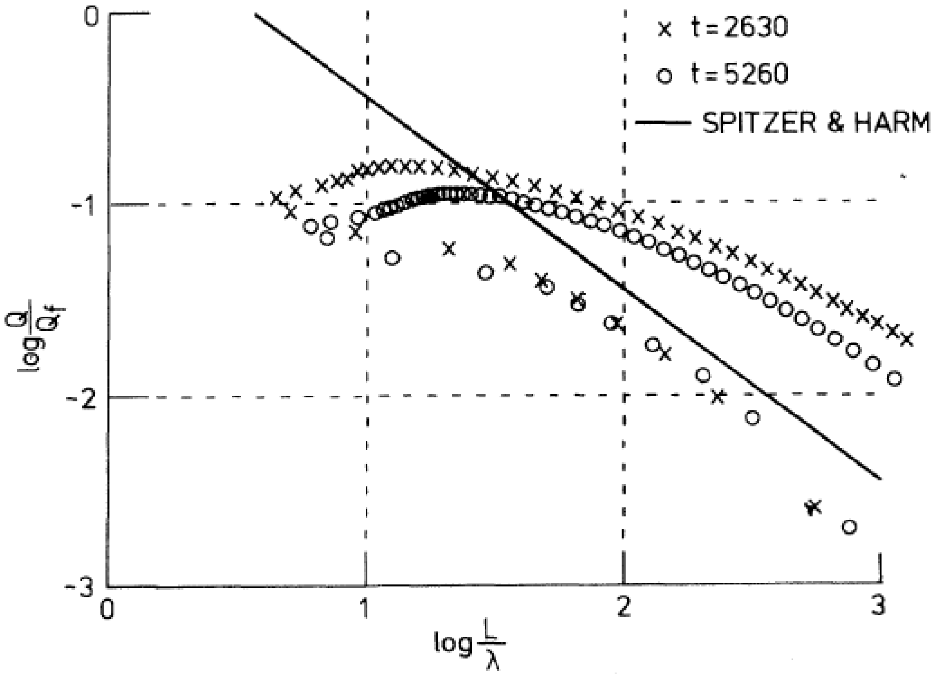
Fokker-Planck account of ‘flux limitation’

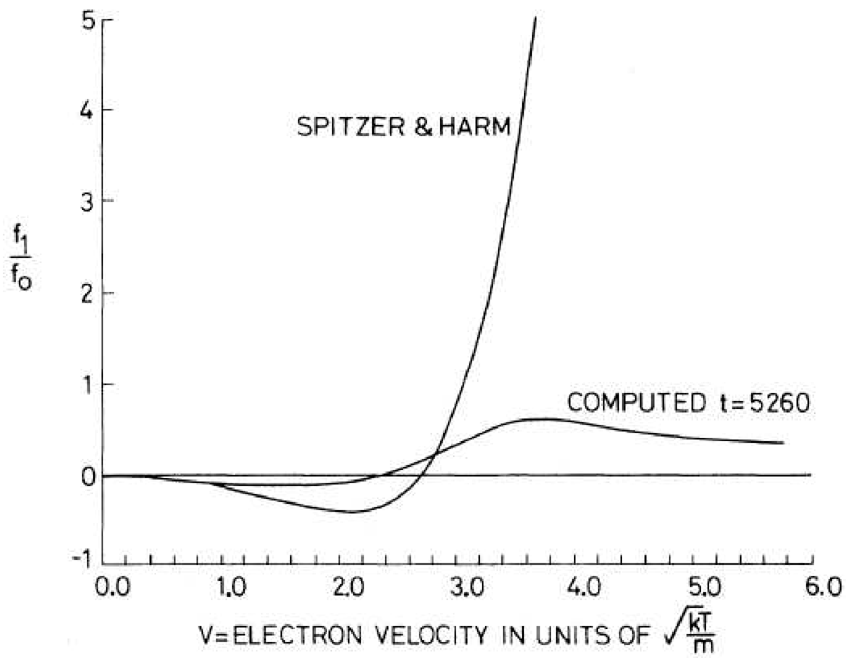
Bell, Evans & Nicholas (1981)

$$\frac{\partial f}{\partial t} + v_x \frac{\partial f}{\partial x} + E\,\frac{\partial f}{\partial v_x} = \left(\frac{\partial f}{\partial t}\right)_{\text{collision}},$$

$$\frac{\partial E}{\partial x} = r\left[\int f\, d^3v - 1\right].$$







Shock ignition

- Betti et al PRL 98 155001 (2007)
- Theobald et al Phys Plasmas15 056306 (2008)
- Ribeyre et al PPCF 51, 015013 (2009)

Stage 1: compression

Stage 2: ignition

High intensity ~6x10^{15}Wcm^{-2}
~100-300psec

Convergent shock heats
fuel to ignition temperature
further compression

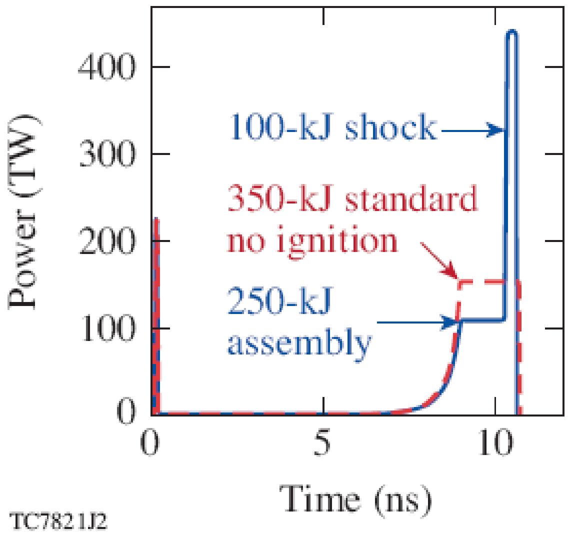
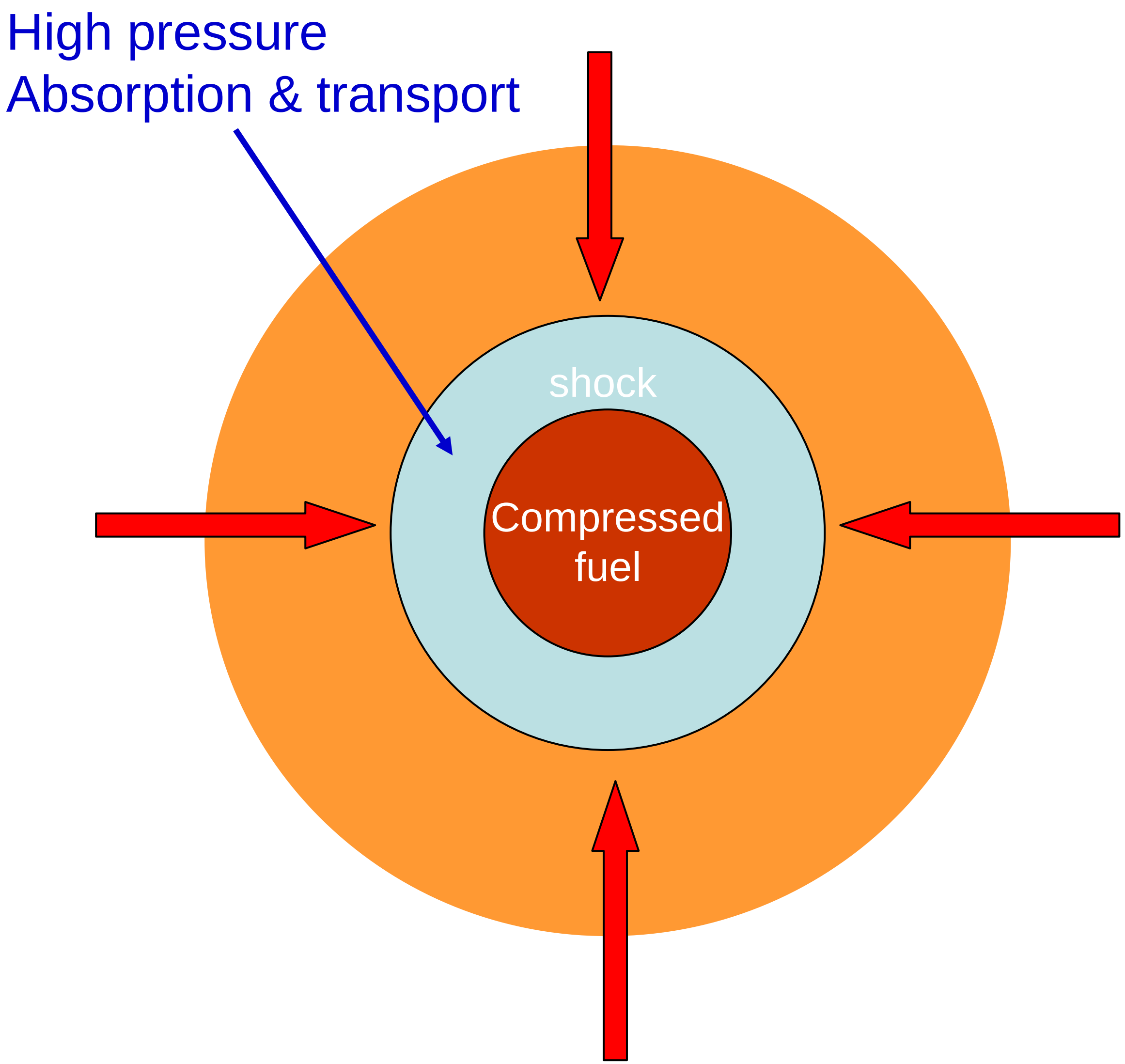
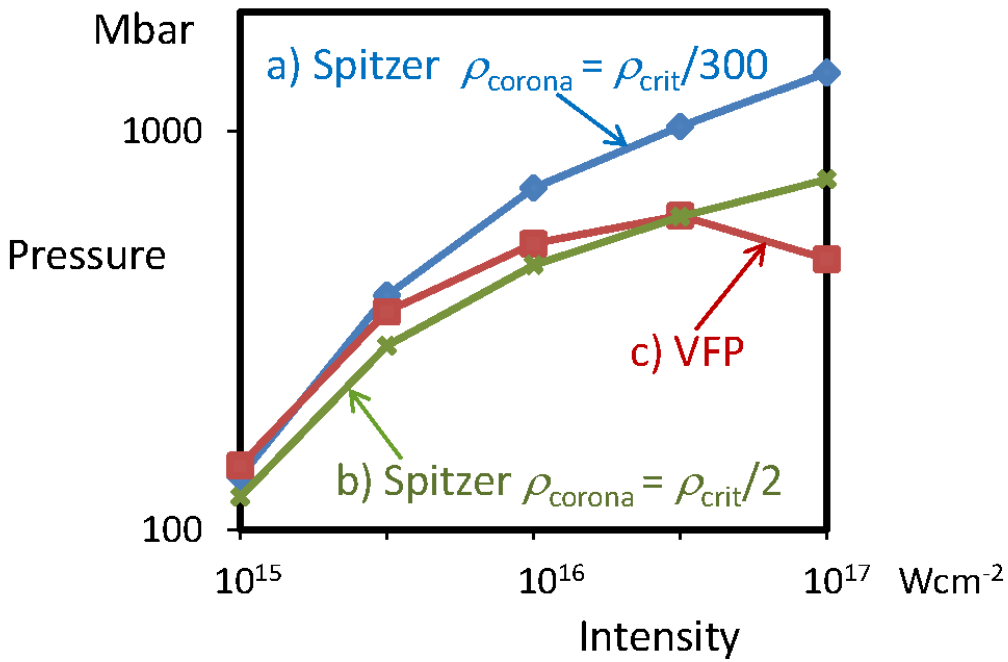


Figure from: Betti et al (2008) JPhys conf series 112 022024



VFP reduces pressure



High energy transport regime

High laser intensity

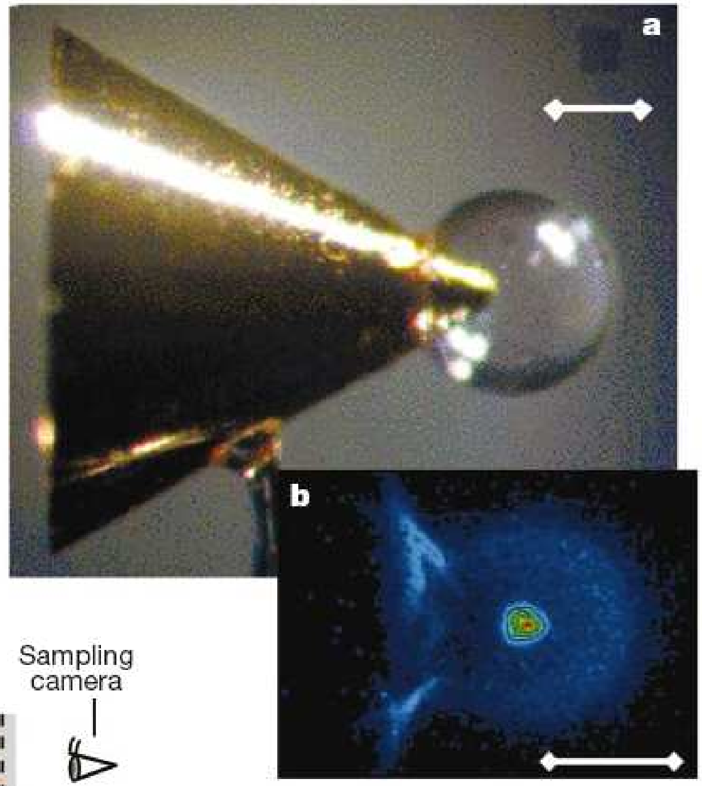
Fast electrons

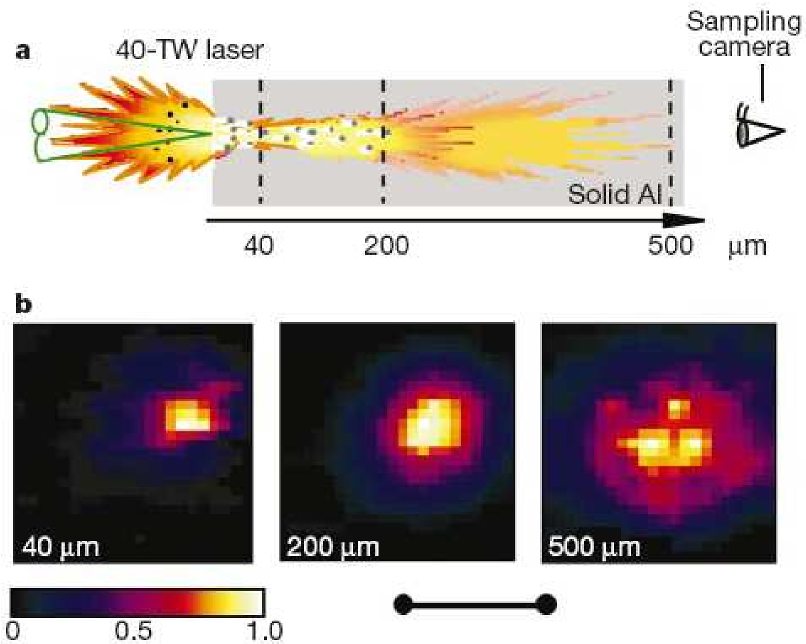
High intensity

Fast electrons

Fast heating of ultrahigh-density plasma as a step towards laser fusion ignition

R. Kodama*, P. A. Norreys†, K. Mima*, A. E. Dangor‡, R. G. Evans§, H. Fujita*, Y. Kitagawa*, K. Krushelnick‡, T. Miyakoshi*, N. Miyanaga*, T. Norimatsu*, S. J. Rose†, T. Shozaki*, K. Shigemori*, A. Sunahara*, M. Tampo*, K. A. Tanaka*||, Y. Toyama*, T. Yamanaka* & M. Zepf‡





Sample collision times & mfp

Conventional ICF

Thermal (1keV) ICF electron at critical density (λ=0.3μm), Z=4, logΛ=4

$$mfp \cong 4\mu m \qquad \tau_e \cong 0.1\text{psec}$$

Thermal (100eV) ICF electron at solid density ($n=30n_{crit}$), Z=4, logΛ=4

$$mfp \cong 0.001\mu m \qquad \tau_e \cong 0.1\text{fsec}$$

Fast ignition

Fast electrons (100keV) at solid density ($n=30n_{crit}$), Z=4, logΛ=4

$$mfp \cong 1000\mu m \qquad \tau_e \cong 3\text{psec}$$

Yet collisions are crucial in Fast Ignition

Collisionality

electron energy in keV

$$mfp = 4\frac{\varepsilon_{keV}^2}{n/n_{crit}}\,\mu m$$

One experiment can cover a range of /energies from eV to MeV

Even when T=1keV,

energy carrying electrons have energy of 10keV

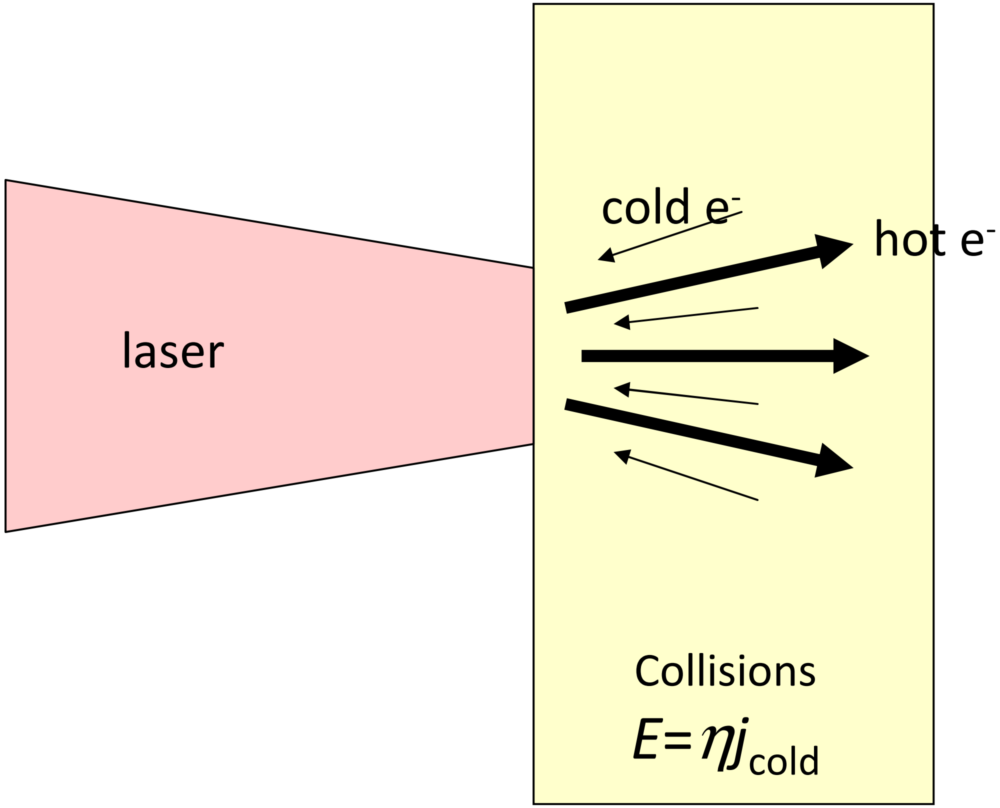
with mfp~100x thermal mfp

Most laser-plasma experiments span range from collisionless to collision-dominated

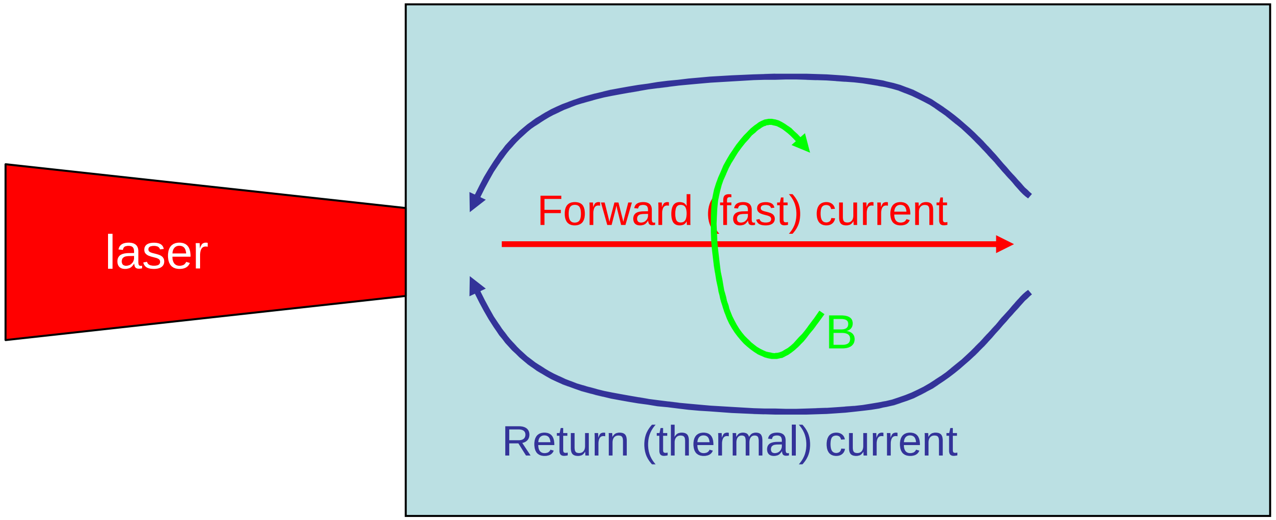
The return current

For quasi-neutrality: $\nabla . j_{hot} = -\nabla . j_{cold}$

Inductance: $j_{hot} = -j_{cold}$ to good approximation



Suppose return current by separate route



Current of 3×10^{9} Amp to carry 30kJ in 100 psec

Beamwidth of 100μm: B ~ 10^{11} G

Energy in magnetic field ~ 1GJ

Energetically impossible for forward/return currents to separate

For quasi-neutrality: $\nabla . j_{hot} = -\nabla . j_{cold}$

Inductance: $j_{hot} = -j_{cold}$ to good approximation

Equation for B

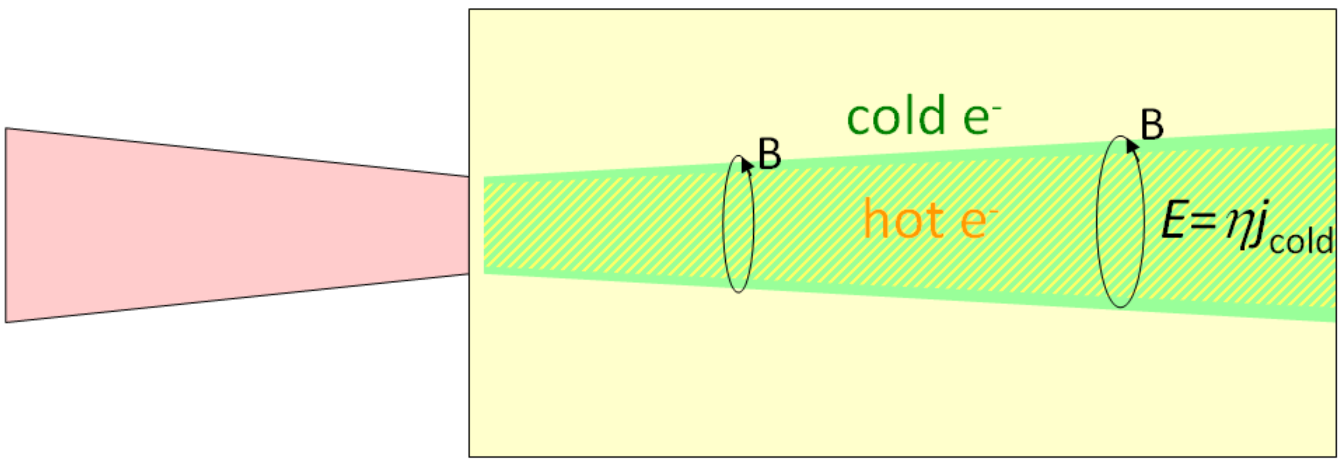
$$\frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\eta \mathbf{j}_{hot}) - \nabla \times \left(\frac{\eta}{\mu_0} \nabla \times \mathbf{B} \right)$$

source — diffusion of B

$\nabla \times B = \mu_0 (j_{hot} + j_{cold})$

$\delta B / \delta t = -\nabla \times E$

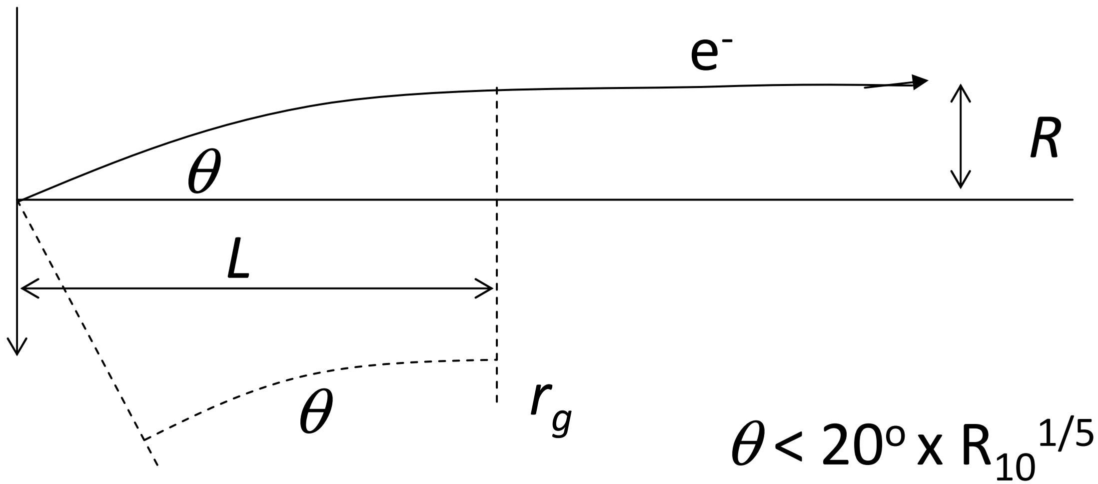
$E = \eta j_{cold}$



cold e⁻ return over slightly larger radius

$B \sim R_{10}^{-3/5} P_{14}^{-1/5} T_{10}^{1/5} t_{12}^{2/5} Z_4^{2/5}$ 1.6 MG

Condition for beam collimation



$$R \approx \theta L$$

$$L \approx \theta r_g$$

$$\theta < 20^o \times R_{10}^{1/5}\, P_{14}^{-1/10}\, T_{10}^{-2/5}\, t_{12}^{1/5}\, Z_4^{1/5}$$

where

R_{10} = beam radius/10μm

P_{14} = power in electron beam/10^{14}W

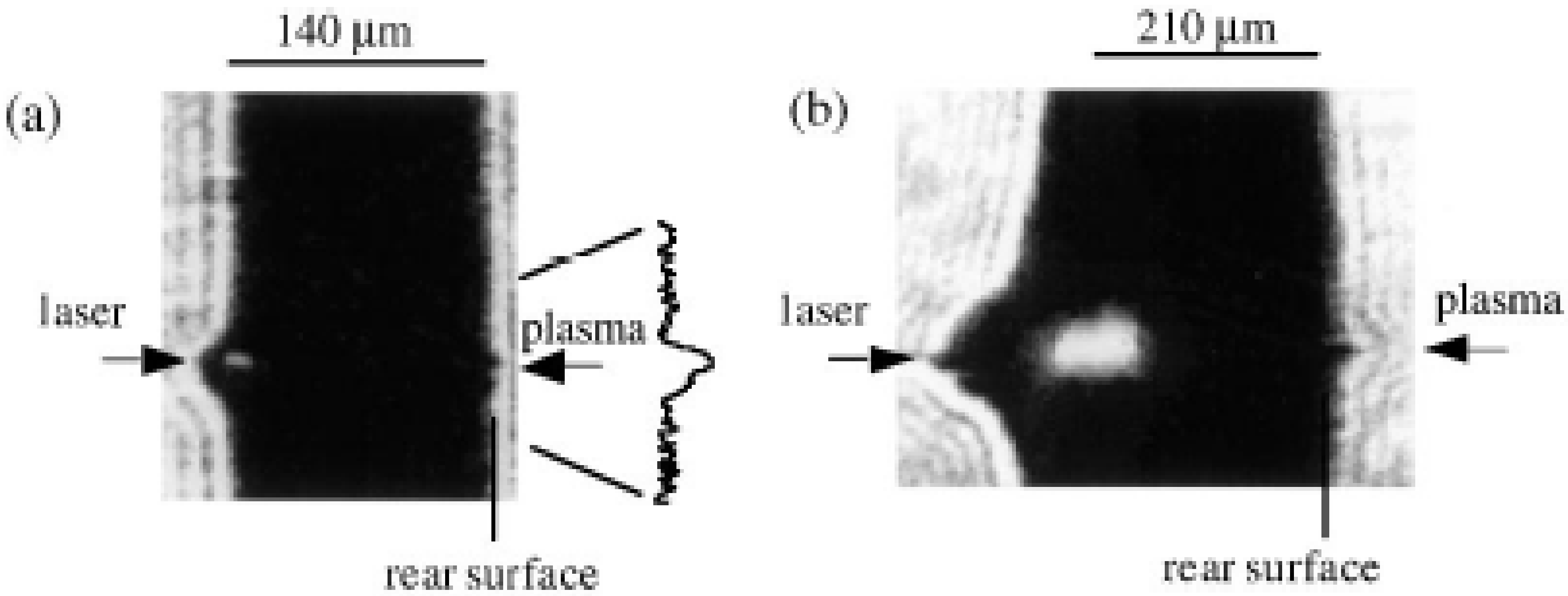
T_{10} = hot electron temp/10MeV

t_{12} = time/psec

Z_4 = ion charge/4

θ_{20} = opening half angle of beam/20^0

Collimated energy transport



Takarakis et al, PRL 81, 999 (1998)

Channel seen through glass

Borghesi et al
PRL 83, 4309 (1999)

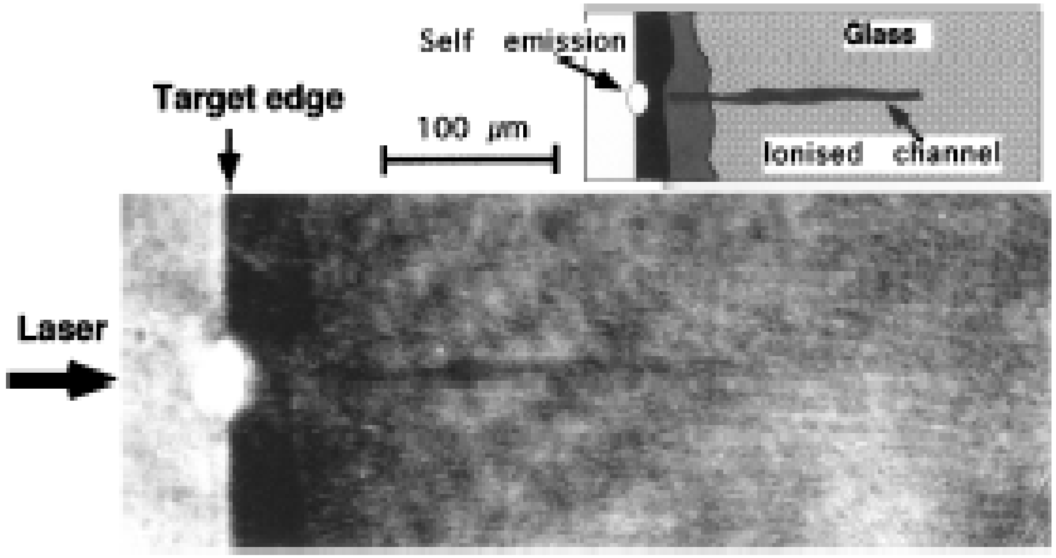


FIG. 2. Shadowgram taken during the interaction of a 20 TW, 1 ps pulse with a solid glass target coated with 1 μm of Al.

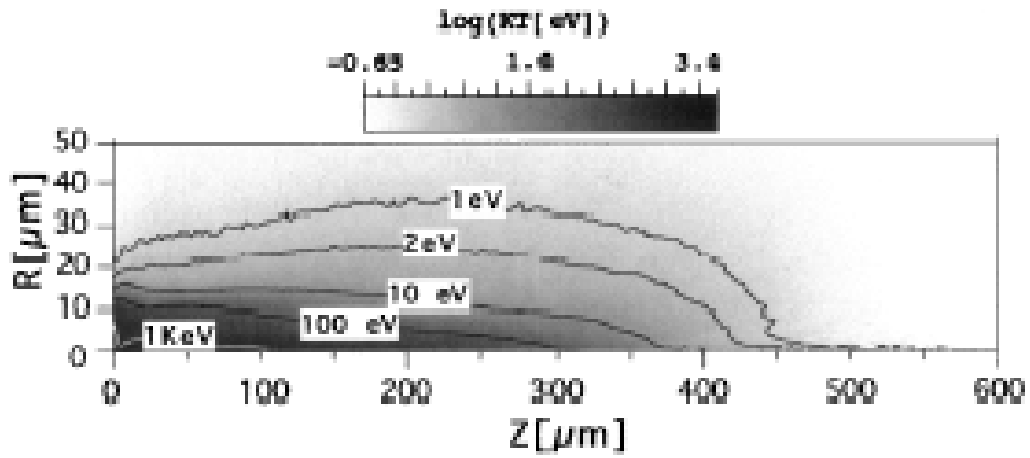


FIG. 4. Profile of background heating in eV as predicted by the hybrid code at 2 ps after the peak of the pulse.

Methods
of
Numerical simulation

Simulation methods

Non-Spitzer nearly-thermal transport, shock ignition

VFP codes ideal

Fast electron transport

VFP accurate but slow & difficult to use for fast ignition (as yet)

Collisionless **PIC** misses essential physics

Hybrid codes easy but thermal resistivity questionable

Collisional PIC not well-suited to collision-dominated regime

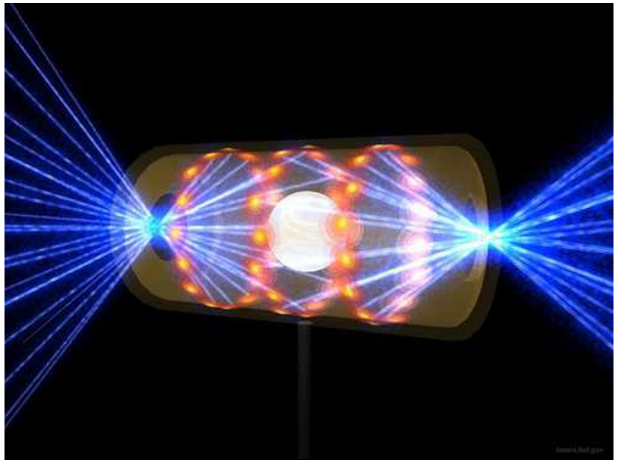
PIC+resistive fields look very promising

Our aim:

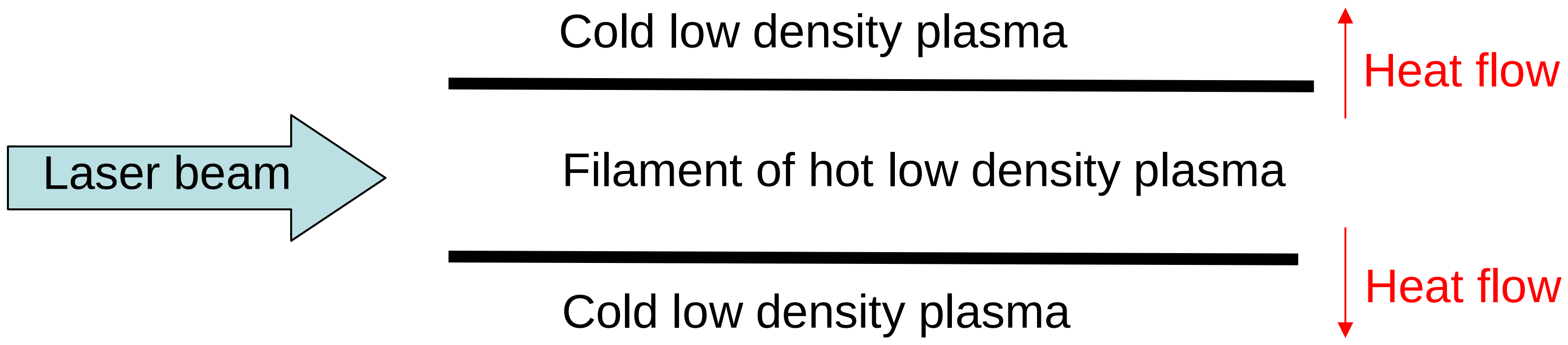
VFP code which can operate across complete range: collisionless/collisional
(Tzoufras et al, submitted to JCompPhys)

Other places where VFP useful: hohlraums

magnetic field
non-local transport (long mfp)



Other places where VFP useful: filamentation



Laser absorption heats plasma in filament

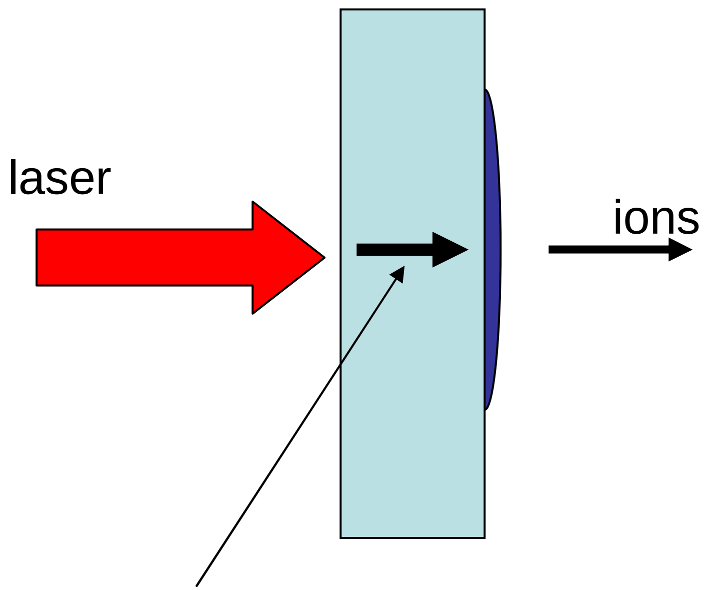
→ High pressure in filament pushes plasma out of filament

→ Refraction focuses laser beam into cavity

→ raises laser intensity in filament

Epperlein 1990:
Non-local transport reduces heat loss from filament
Gives stronger filamentation

Other places where VFP useful: ion acceleration

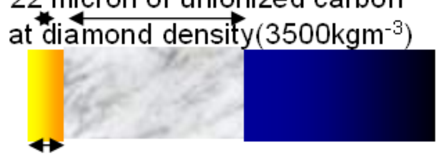


Fast electrons form electric sheath at rear surface
Electric sheath field accelerates ions

Fast electrons drive resistive magnetic field – collimated transport

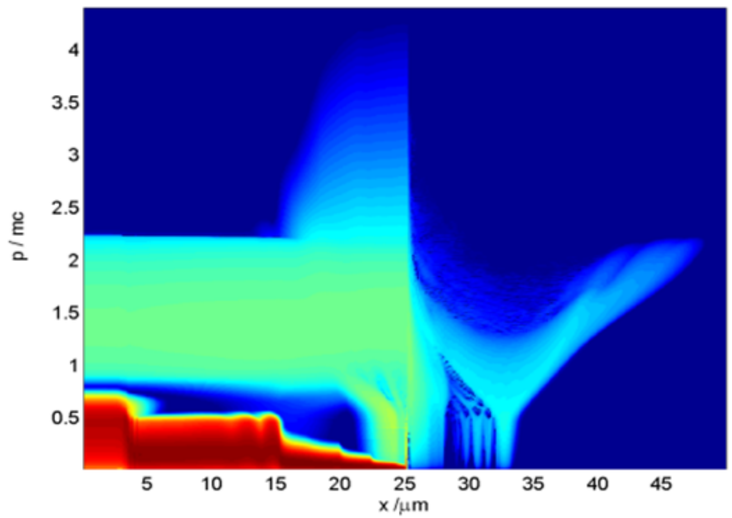
Other places where VFP useful: ionisation

VFP a platform for extra physics

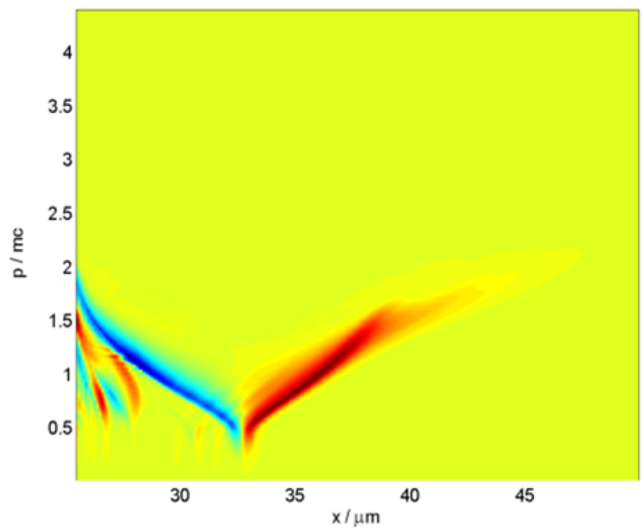


Ionisation included by Alex Robinson

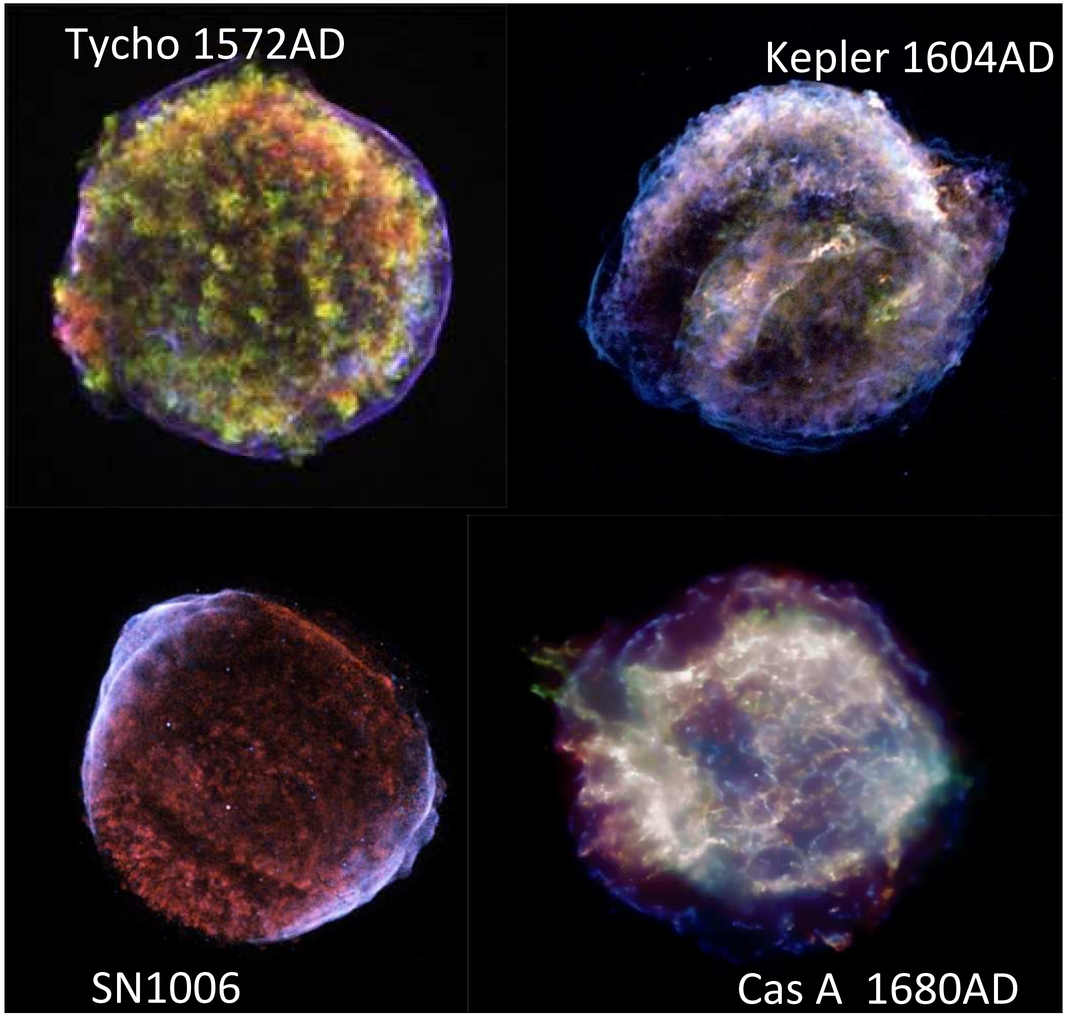
Electron density in phase space



Electron flux



Other places where VFP useful: astrophysics



Chandra observations

NASA/CXC/Rutgers/ J.Hughes et al.

NASA/CXC/Rutgers/ J.Warren & J.Hughes et al.

NASA/CXC/NCSU/ S.Reynolds et al.

NASA/CXC/MIT/UMass Amherst/ M.D.Stage et al.

How VFP compares with other simulation methods

Equations representing plasmas

Particle in cell PIC

$$\frac{dp}{dt} = e(E + v \times B) \qquad \frac{dr}{dt} = v$$

$$\frac{\partial B}{\partial t} = -\nabla \times E \qquad \frac{\partial E}{\partial t} = c^2 \nabla \times B - \frac{j}{\varepsilon_0}$$

Fluid/MHD

$$\frac{d\rho}{dt} = -\nabla.(\rho u) \qquad \rho \frac{du}{dt} = -\nabla P - \frac{1}{\mu_0} B \times (\nabla \times B)$$

$$\frac{dU}{dt} = -P\nabla.u + \nabla.(\kappa \nabla T) \qquad P = P(U, \rho) \qquad \frac{\partial B}{\partial t} = \nabla \times (u \times B)$$

How do these equations simulate the same configuration?

Apply in different limits

PIC/MHD comparison

Thermal relaxation

MHD assumes thermal relaxation
PIC makes no assumptions about momentum distribution

Collisions

MHD best when collision-dominated
PIC best when collisionless

Material properties

MHD models equation of state, transport properties, radiation transfer
PIC neglects these

Timescales

PIC: laser/Langmuir frequency, models absorption LPI instabilities
MHD: models target implosion, hydro instabilities

VFP in the middle

Thermal relaxation: non-Maxwellian but simply structured distributions

Collisions: range between dominant and weak

Material properties: included through fluid treatment of ions

Timescales: kinetic theory on hydro timescale

The Vlasov-Fokker-Planck (VFP) equation

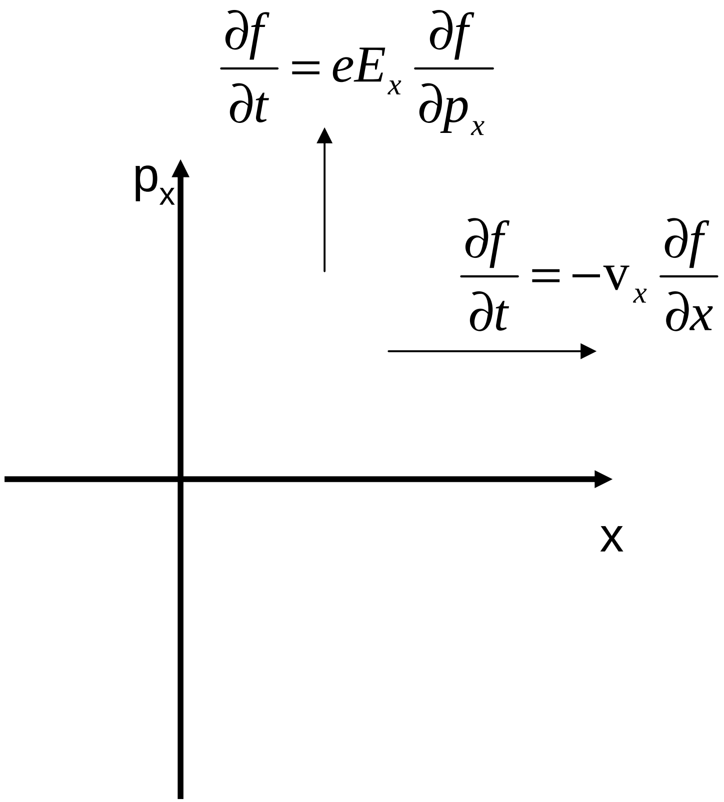
$$\underbrace{\frac{\partial f}{\partial t} + \mathbf{v}.\frac{\partial f}{\partial \mathbf{r}} - e\left(\mathbf{E} + \mathbf{v} \times \mathbf{B}\right).\frac{\partial f}{\partial \mathbf{p}}}_{\text{Vlasov equation (collisioness)}} = \underbrace{C(f)}_{\text{Collisions Fokker-Planck}}$$

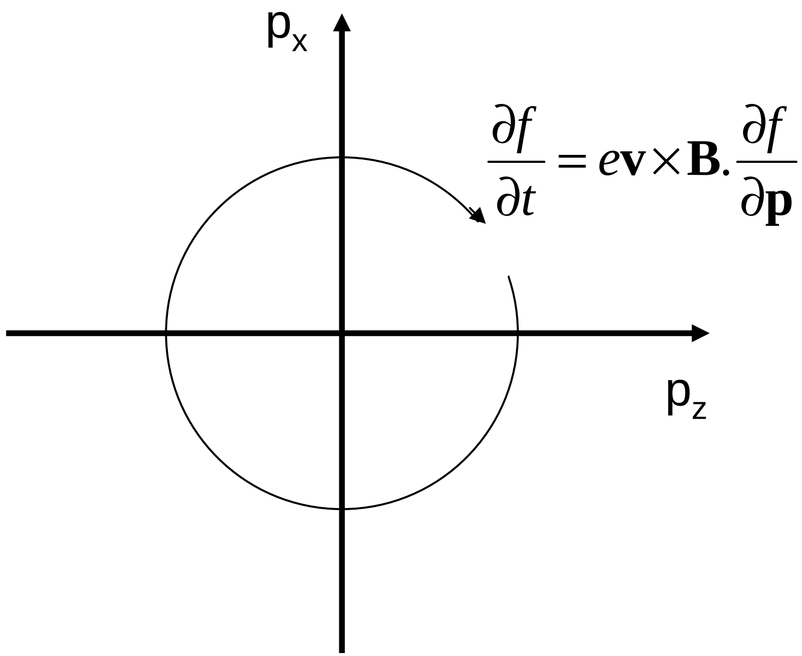
$$f(x, y, z, p_x, p_y, p_z, t)\, dx\, dy\, dz\, dp_x\, dp_y\, dp_z$$

= number of electrons in phase space volume $dx\, dy\, dz\, dp_x\, dp_y\, dp_z$

Vlasov equation (no collisions)

Vlasov code solves for *f* in (**r**,**p**) space $\frac{\partial f}{\partial t} + \mathbf{v}.\frac{\partial f}{\partial \mathbf{r}} - e\left(\mathbf{E} + \mathbf{v}\times\mathbf{B}\right).\frac{\partial f}{\partial \mathbf{p}} = 0$





In (x,p_x,p_z) natural to use square grid
but magnetic field gives diagonal motion

Fokker-Planck equation for collisions

Small angle scattering: diffusion & advection in momentum space

$$C(f) = \frac{\partial}{\partial \mathbf{p}} . \left(\underset{\approx}{\mathbf{D}} . \frac{\partial f}{\partial \mathbf{p}} - \mathbf{A} f \right)$$

Diffusion tensor

Advection in momentum space
eg slowing down

Diffusion + advection = ‘Fokker-Planck’ equation

Collisions (Fokker-Planck)

Collisions generate entropy by diffusion, reduce information
Use to advantage

Dominant collision terms in spherical polars in momentum space

$$\frac{\partial f}{\partial t} = \underbrace{\frac{1}{p^2}\frac{\partial}{\partial p}\left(p^2 D_p \frac{\partial f}{\partial p}\right) - \frac{1}{p^2}\frac{\partial}{\partial p}\left(p^2 A f\right)}_{\text{Balance these to get Maxwellian}} + \underbrace{\frac{1}{p^2 \sin\vartheta}\frac{\partial}{\partial \vartheta}\left(\sin\vartheta\, D_\perp \frac{\partial f}{\partial \vartheta}\right) + \frac{D_\perp}{p^2 \sin^2\vartheta}\frac{\partial^2 f}{\partial \phi^2}}_{\text{Angular scattering, mainly by ions}}$$

Natural expansion for *f*

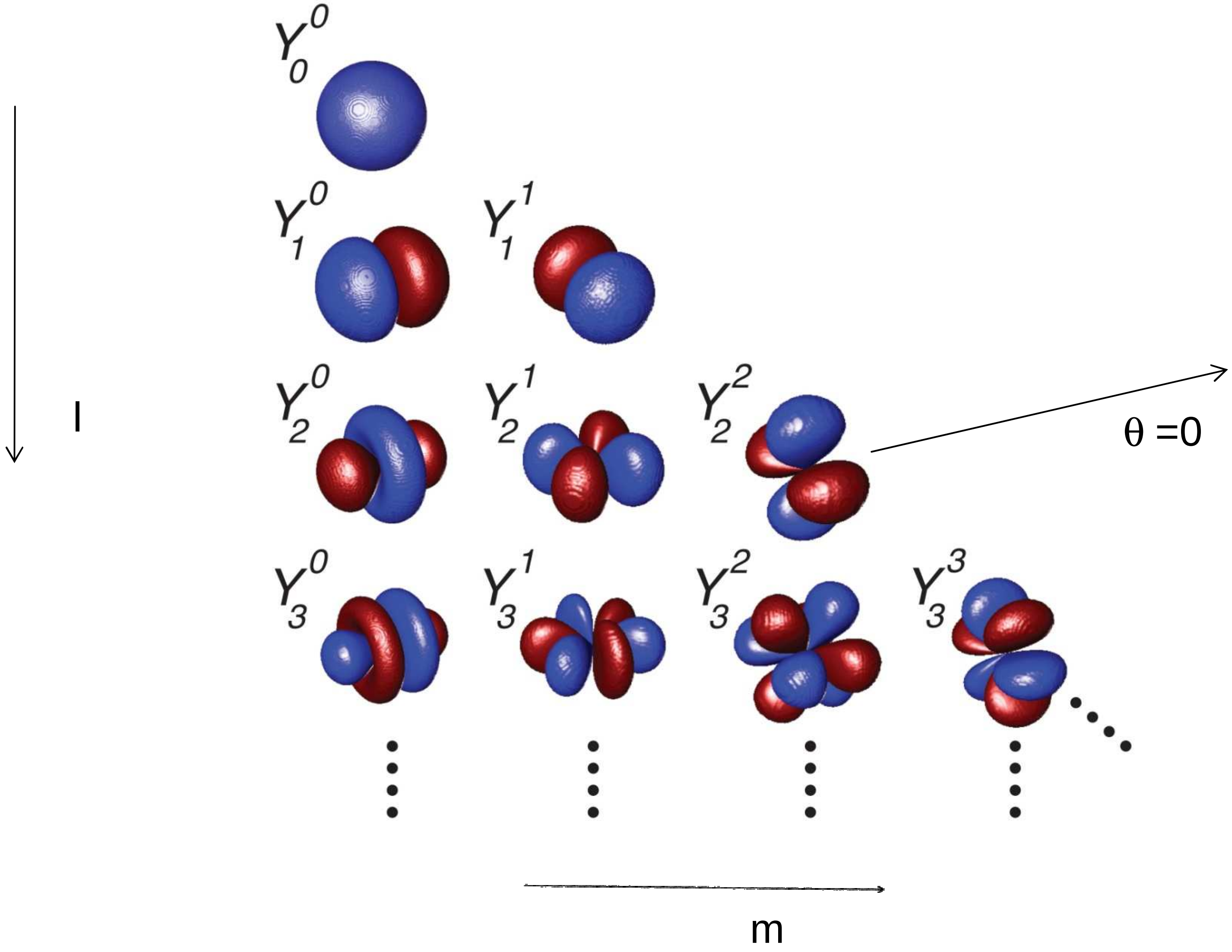
$$f(x,\mathbf{p},t) = \sum_{l,m} f_l^m(x,p,t) P_l^m(\cos\theta) e^{im\phi}$$

Spherical harmonics

Terminate expansion at low order

saves memory, processor time

Spherical harmonics



In spherical polars

$$\frac{\partial f}{\partial t} = \frac{1}{p^2}\frac{\partial}{\partial p}\left(p^2\left[D_p \frac{\partial f}{\partial p} - Af\right]\right) + \frac{1}{p^2 \sin\vartheta}\frac{\partial}{\partial \vartheta}\left(\sin\vartheta\, D_\perp \frac{\partial f}{\partial \vartheta}\right) + \frac{D_\perp}{p^2 \sin^2\vartheta}\frac{\partial^2 f}{\partial \phi^2}$$

Angular scattering stronger than energy diffusion $D_\perp \approx Z D_p$

Simplifies to $\frac{\partial f_l^m}{\partial t} = \frac{1}{p^2}\frac{\partial}{\partial p}\left(p^2\left[D_p \frac{\partial f_l^m}{\partial p} - A f_l^m\right]\right) - \frac{l(l+1)}{2}\frac{D_\perp}{p^2} f_l^m$

Dominant term

Angular scattering $\frac{\partial f_l^m}{\partial t} = -\frac{l(l+1)}{2}\frac{D_\perp}{p^2} f_l^m$ has solution $f_l^m \propto \exp\left(-\frac{l(l+1)}{2}\frac{D_\perp}{p^2} t\right)$

High order harmonics decay rapidly – can truncate harmonic expansion

Other ways of gridding momentum space

Rectangular grid in p_x, p_y p_z

OK in 1D but too much memory/computation in 3D
Collisions move diagonally across grid

Grid in cos θ

need many grid points for simple function – eg 1st harmonic
difficult boundary at θ=0 and $\theta=\pi$

Grid in ϕ

fine for collisionless problems in special geometry

Tensor expansion

$$f = \sum f_0(p) + f_i(p)\frac{p_i}{p} + f_{ij}(p)\frac{p_i p_j}{p^2} + f_{ijk}(p)\frac{p_i p_j p_k}{p^3} + f_{ijkl}(p)\frac{p_i p_j p_k p_l}{p^4} \ldots$$

Equivalent to spherical harmonics

Simpler to first order $$f = f_0(p) + f_x(p)\frac{p_x}{p} + f_y(p)\frac{p_y}{p} + f_z(p)\frac{p_z}{p}$$

Messy at high order

Limitations of spherical polars

Deciding where to truncate the expansion

truncating too early can cause run to collapse with positive or negative spikes in f

present methods do not guarantee positivity

electric field can react badly

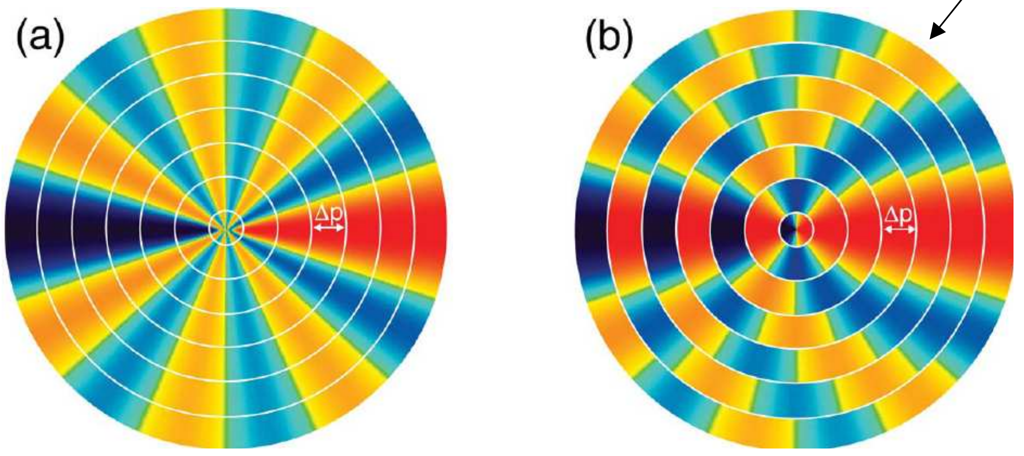
Can use ‘artificial collisions’ to reduce anisotropy

Singularity at p=0

truncate early close to p=0

Phase change rotates about centre

(Michail Tzoufras)



Collisions remove anisotropy at p=0

Magnetic field in spherical polars

Magnetic field rotates in momentum

For example: ‘Vertical’ B rotates l=10, m=0

General formula for effect of magnetic field:

$$\mathcal{B}_n^m = -\mathrm{i}\frac{eB_x}{m_e} m f_n^m - \frac{1}{2}\frac{e}{m_e}[(n-m)(n+m+1)(B_z - \mathrm{i}B_y) f_n^{m+1} - (B_z + \mathrm{i}B_y) f_n^{m-1}].$$

For $m = 0$,

$$\Re[\mathcal{B}_n^0] = -\frac{e}{m_e}\, n(n+1)(B_z \Re[f_n^1] + B_y \Im[f_n^1])$$

Algebraic: no differentials

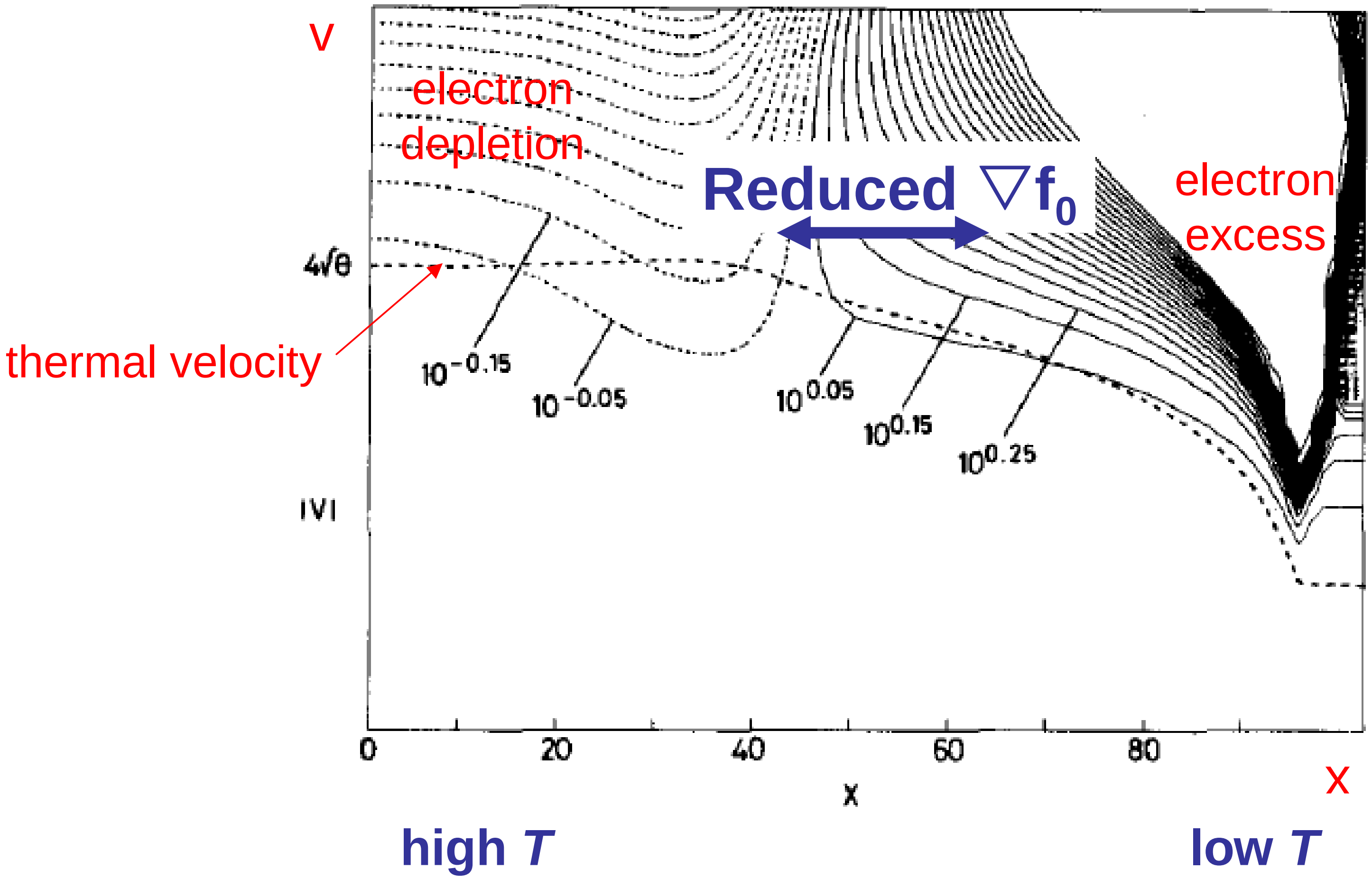
$eB/m_e = 2\times10^{13}\,\mathrm{sec}^{-1}$ when B=1MG

Solve implicitly as complex tri-diagonal matrix equation

Diffusion approximation

where VFP really wins over other methods
(when it is valid)

The essence of non-local transport



Diffusion approximation

Otherwise known as ‘f_0+f_1‘ approximation

Tensor form

$$f = f_0(p) + f_x(p)\frac{p_x}{p} + f_y(p)\frac{p_y}{p} + f_z(p)\frac{p_z}{p} \iff f = f_0 + \mathbf{f}_1 \cdot \frac{\mathbf{p}}{|\mathbf{p}|}$$

Spherical harmonic

$$f = f_0 + f_1^0(p)\cos\vartheta + \mathrm{Re}\left\{f_1^1(p)\right\}\sin\vartheta\cos\phi - \mathrm{Im}\left\{f_1^1(p)\right\}\sin\vartheta\sin\phi$$

Assume (correct in many cases) that higher order terms are negligible

This is where VFP is a real winner!

Diffusion approximation

Use vector notation

$$f(\mathbf{p}) = f_0(|\mathbf{p}|) + \mathbf{f}_1(|\mathbf{p}|).\frac{\mathbf{p}}{|\mathbf{p}|} \quad \text{where} \quad \mathbf{f}_1 = \left(f_x, f_y, f_z\right)$$

f_0 & $\mathbf{f}_1$ are functions of magnitude of momentum

First two moment equations

Number conservation:

$$\frac{\partial f_0}{\partial t} + \frac{\mathrm{v}}{3}\nabla.\mathbf{f}_1 = C_0\left(f_0, f_0\right)$$

Divergence of flux

Collisional energy exchange

Momentum conservation

$$\frac{\partial \mathbf{f}_1}{\partial t} + \mathrm{v}\nabla f_0 - e\mathbf{E}\frac{\partial f_0}{\partial p} - \boldsymbol{\omega}\times\mathbf{f}_1 = C_1\left(f_0, \mathbf{f}_1\right) - \boldsymbol{\nu}\mathbf{f}_1$$

Pressure gradient

Electric field
Maintains neutrality

Rotation by mag field
$\omega = e\mathbf{B}/m$

Angular scattering
by collisions

Simplest VFP code (B = 0)

Set up computational grid in x and v

Define f_0 on grid

In finite difference form, solve

$$\frac{\partial f_0}{\partial t} = \frac{\partial}{\partial x}\left(\frac{\mathrm{v}^2}{3\nu}\frac{\partial f_0}{\partial x}\right) - \frac{\partial}{\partial x}\left(\frac{\mathrm{v}eE}{3\nu}\frac{\partial f_0}{\partial \mathrm{v}}\right) + \frac{1}{\mathrm{v}^2}\frac{\partial}{\partial \mathrm{v}}\left(\mathrm{v}^2\left[D_p\frac{\partial f_0}{\partial \mathrm{v}} - Af_0\right]\right)$$

$$D_p \propto \frac{1}{\mathrm{v}^3}\int_0^{\mathrm{v}} u^2 F(u)\,du \qquad A \propto \frac{1}{\mathrm{v}^2}\int_0^{\mathrm{v}} u f_0(u)\,du \qquad F(u) = \int_u^{\infty} w f_0\,dw$$

Calculate heat flow and electrical current by integrating in velocity

$$Q = -\frac{2\pi m_e}{3}\int \frac{\mathrm{v}^6}{\nu}\frac{\partial f_0}{\partial x}d\mathrm{v} + \frac{2\pi eE}{3}\int \frac{\mathrm{v}^5}{\nu}\frac{\partial f_0}{\partial \mathrm{v}}d\mathrm{v}$$

$$\mathrm{j} = -\frac{4\pi e}{3}\int \frac{\mathrm{v}^4}{\nu}\frac{\partial f_0}{\partial x}d\mathrm{v} - \frac{2\pi e^2 E}{3m_e}\int \frac{\mathrm{v}^3}{\nu}\frac{\partial f_0}{\partial \mathrm{v}}d\mathrm{v}$$

Calculate E from

$$\frac{\partial E}{\partial t} = -\frac{j}{\varepsilon_0}$$

Dominant terms with magnetic field

$$\frac{\partial f_0}{\partial t} + \frac{\mathrm{v}}{3}\nabla.\mathbf{f}_1 = \frac{1}{p^2}\frac{\partial}{\partial p}\left(p^2\left[D_p\frac{\partial f_0}{\partial p} - Af_0\right]\right)$$

$$\mathrm{v}\nabla f_0 - e\mathbf{E}\frac{\partial f_0}{\partial p} - \boldsymbol{\omega}\times\mathbf{f}_1 = -\boldsymbol{\nu}\mathbf{f}_1$$

Even in one spatial dimension, $\mathbf{f}_1$ can have three components

$$\mathbf{f}_1 = -\frac{1}{\omega^2+\nu^2}\left\{\nu\boldsymbol{\sigma} + \boldsymbol{\omega}\times\boldsymbol{\sigma} + \frac{\boldsymbol{\omega}(\boldsymbol{\omega}.\boldsymbol{\sigma})}{\nu}\right\}$$

where

$$\boldsymbol{\sigma} = \mathrm{v}\nabla f_0 - e\mathbf{E}\frac{\partial f_0}{\partial p}$$

Heat flow down temperature gradient

Heat flow along magnetic field

Heat flow perpendicular to temperature gradient & magnetic field

VFP solved implicitly

Similar to solving diffusion equations implicitly

Implicit solution for electric field

$$\frac{\partial \mathbf{E}}{\partial t} = -\frac{\mathbf{j}}{\varepsilon_0} \quad \left(+ c^2 \nabla \times \mathbf{B} \right)$$

Electron plasma oscillations at solid density: ω_{pe}^{-1}~0.3fsec

For numerical stability Δt ~ 0.1fsec

Implicit solution: $$\frac{E_i^{new} - E_i^{old}}{\Delta t} = \frac{j_i^{new}}{\varepsilon_0}$$

Δt limited by accuracy, not stability

Integrates over plasma oscillations (inessential)

But j^{new} depends on f^{new} and E^{new}

Implicit solution for both f^{new} and E^{new} cannot be decoupled

Implicit solution – large non-linear matrix equation

$$\frac{f_0^{new} - f_0^{old}}{\Delta t} = \frac{\partial}{\partial x}\left(\frac{\mathrm{v}^2}{3\nu}\frac{\partial f_0^{new}}{\partial x}\right) - \frac{\partial}{\partial x}\left(\frac{\mathrm{v}eE^{new}}{3\nu}\frac{\partial f_0^{new}}{\partial \mathrm{v}}\right) + \frac{1}{\mathrm{v}^2}\frac{\partial}{\partial \mathrm{v}}\left(\mathrm{v}^2\left[D_p\frac{\partial f_0^{new}}{\partial \mathrm{v}} - Af_0^{new}\right]\right)$$

$$j^{\mathrm{new}} = -\frac{4\pi e}{3}\int\frac{\mathrm{v}^4}{\nu}\frac{\partial f_0^{new}}{\partial x}d\mathrm{v} - \frac{2\pi e^2 E^{new}}{3m_e}\int\frac{\mathrm{v}^3}{\nu}\frac{\partial f_0^{new}}{\partial \mathrm{v}}d\mathrm{v}$$

Iterate on non-linear term

$$\frac{E^{new} - E^{old}}{\Delta t} = \frac{j^{new}}{\varepsilon_0}$$

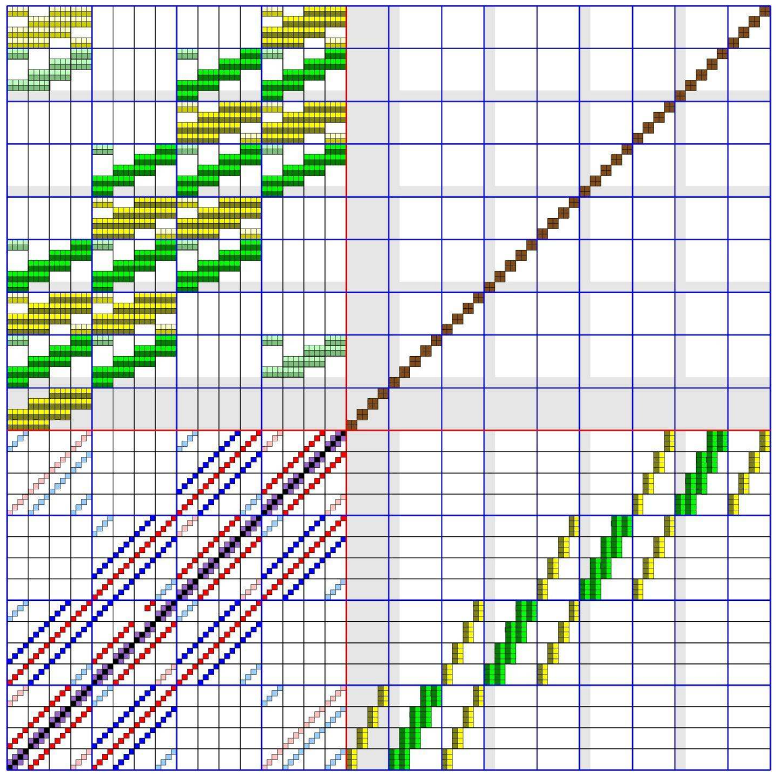
Allows large timestep
Works well for small excursions from equilibrium

IMPACT – a real tour de force!

Implicit code written by Robert Kingham et al (Imperial)

2D, includes magnetic field
Extended to f_2 (Alex Thomas)

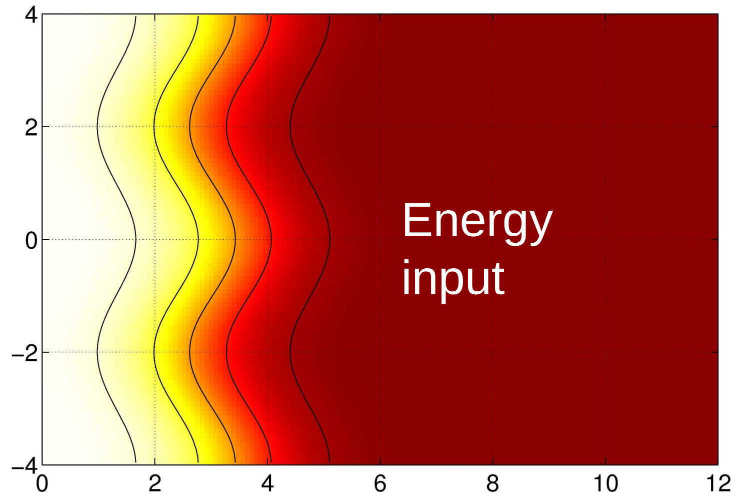
Matrix structure for 4x4x4 grid



Good for
Long pulses
Hohlraums
Magnetic field

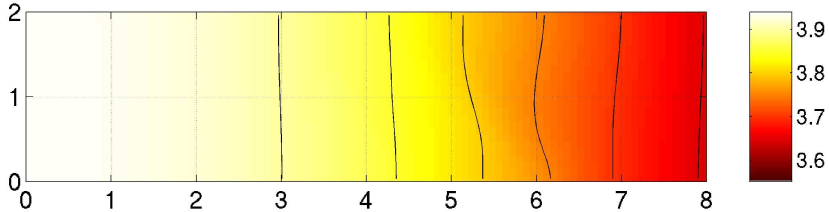
Optimal for leading order non-local effects
Computationally efficient

IMPACT example:
Non-local magnetic field generation
(Kingham et al)

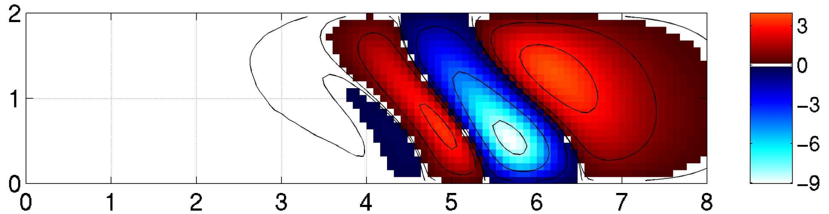


Uniform density

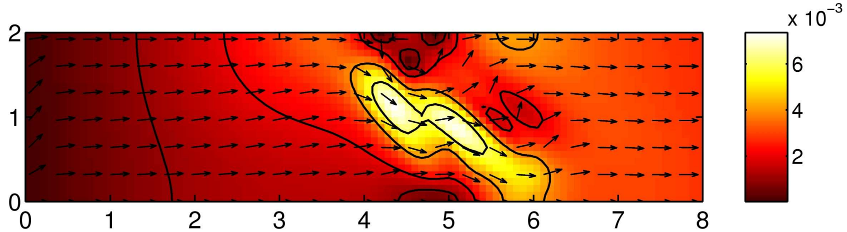
Temperature



Magnetic field



Heat flow



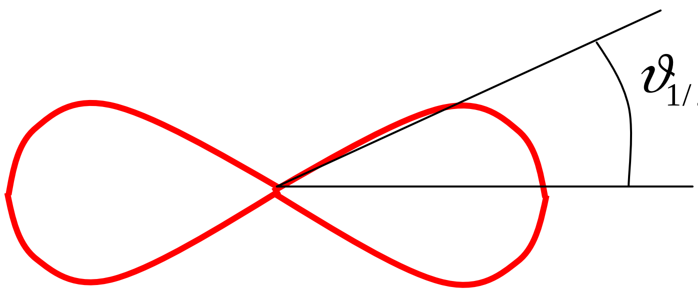
Cases where f_0+f_1 is not enough

f_0+f_1 cannot model beams

$$f(x,\mathbf{p},t)=\sum_{l,m} f_l^m(x,p,t)P_l^m(\cos\theta)e^{im\phi}$$

Expansion to order n gives any polynomial in cosθ to order n

eg $\quad f=\cos^n\vartheta\cong\left(1-\frac{\vartheta^2}{2}\right)^n$

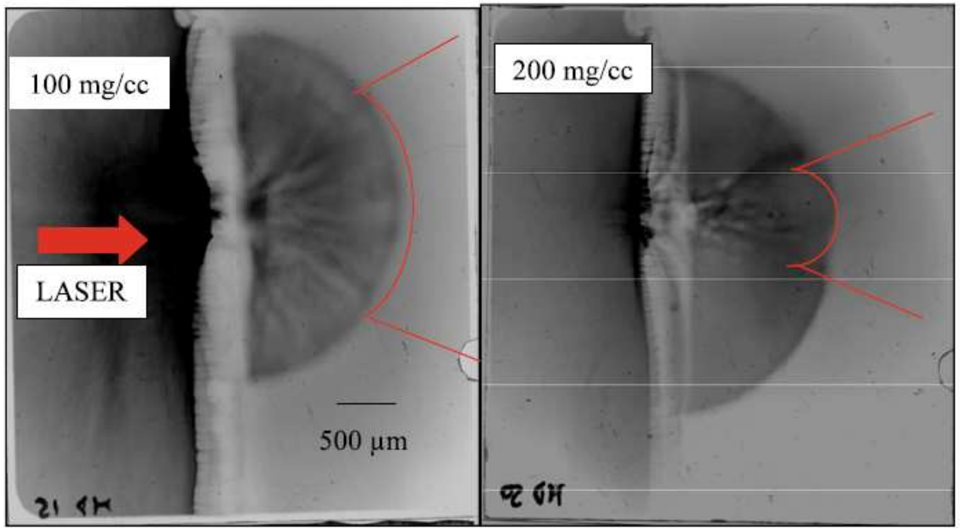


$$\vartheta_{1/2}=\frac{80^{O}}{\sqrt{n}}$$

Need 16 harmonics for beam half-angle 20^{O}

Weibel instability

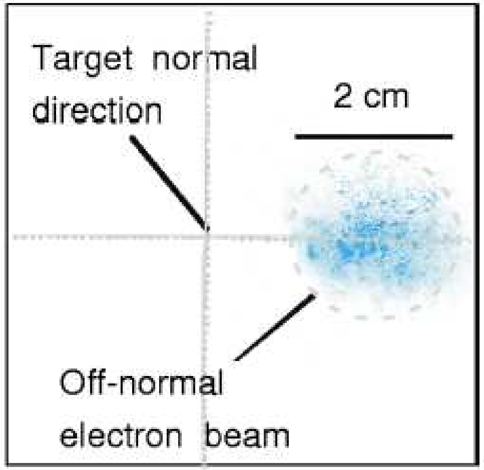
Ramakrishna et al (2009)



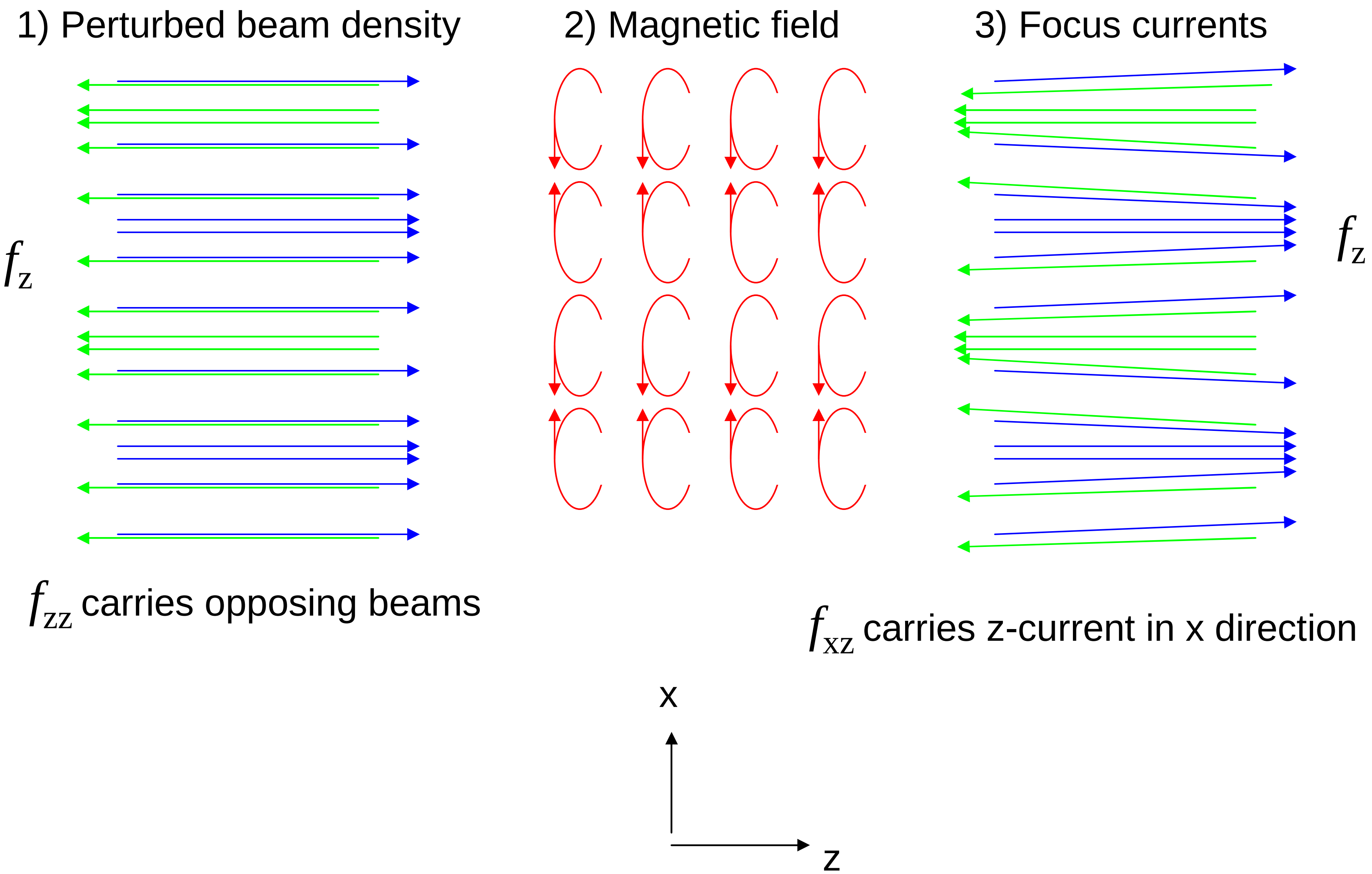
T= 2ps

T=2ps

Wei et al (2004)



Weibel instability: opposing energetic electron beams



Diffusion (f_0+f_1) approximation

Insufficient for

1) strong anisotropy (eg beams)

2) magnetic field varying on scale of Larmor radius
   moving currents in space

3) current filamentation instabilities

If f_2 important then so probably is f_3, f_4 etc

KALOS

AIM: encompass collisional & collisionless in one code

Formulated initially in 2002
Latest version by Michail Tzoufras (2011)

KALOS code

Kinetic
a
Laser-plasma
o
Simulation

Expand velocity dist^n in spherical harmonics

$$f(x,y,v,\theta,\phi,t) = \sum f_{nm}(x,y,v,t)\, P_n^{|m|}(\cos\theta)\, e^{im\phi}$$

velocity coordinates in 3D

- Any degree of anisotropy by expanding to any order
- Operates well as Vlasov code (efficiently)
- Collisions and B easily included
- Collisional damping rate ~ n(n+1)
- Numerical equations simple – efficient despite small explicit timestep

$$f = \sum_{n=0}^{n_{\max}} \sum_{m=-n}^{n} f_n^m(r,z,p) P_n^{|m|}(\cos\vartheta) e^{im\varphi}$$

$$\frac{\partial f_n^m}{\partial t} + \frac{n(n+1)}{2} \upsilon f_n^m - \frac{1}{v^2}\frac{\partial}{\partial v}\left(D_{\parallel} \frac{\partial f_n^m}{\partial v} + E f_n^m \right)$$

$$+\omega_z i \left\{ m f_n^m \right\} + \omega_r \frac{i}{2} \left\{ (n-m)(n+m+1) f_n^{m+1} + f_n^{m-1} \right\} + \omega_\vartheta \frac{1}{2} \left\{ (n-m)(n+m+1) f_n^{m+1} - f_n^{m-1} \right\}$$

$$=$$

$$-\left(\frac{n-m}{2n-1}\right) v \frac{\partial f_{n-1}^m}{\partial z} - \left(\frac{n+m+1}{2n+3}\right) v \frac{\partial f_{n+1}^m}{\partial z}$$

$$-\frac{v}{2} \left\{ \begin{array}{l} \frac{1}{2n-1}\left[r^{m-1} \frac{\partial (r^{-m+1} f_{n-1}^{m-1})}{\partial r} - (n-m)(n-m-1) r^{-m-1} \frac{\partial (r^{m+1} f_{n-1}^{m+1})}{\partial r} \right] \\ + \frac{1}{2n+3}\left[-r^{m-1} \frac{\partial (r^{-m+1} f_{n+1}^{m-1})}{\partial r} + (n+m+1)(n+m+2) r^{-m-1} \frac{\partial (r^{m+1} f_{n+1}^{m+1})}{\partial r} \right] \end{array} \right\}$$

$$-eE_z \left\{ \frac{n-m}{2n-1} G_{n-1}^m + \frac{n+m+1}{2n+3} H_{n+1}^m \right\}$$

$$-eE_r \frac{1}{2} \left\{ \frac{1}{2n-1}\left[G_{n-1}^{m-1} - (n-m)(n-m-1) G_{n-1}^{m+1} \right] + \frac{1}{2n+3}\left[-H_{n+1}^{m-1} + (n+m+1)(n+m+2) H_{n+1}^{m+1} \right] \right\}$$

$$-eE_\vartheta \frac{i}{2} \left\{ \frac{1}{2n-1}\left[-G_{n-1}^{m-1} - (n-m)(n-m-1) G_{n-1}^{m+1} \right] + \frac{1}{2n+3}\left[H_{n+1}^{m-1} + (n+m+1)(n+m+2) H_{n+1}^{m+1} \right] \right\}$$

Spatial advection in cylindrical (r,z) geometry

$$\frac{\partial f_n^m}{\partial t} = -\left(\frac{n-m}{2n-1}\right) v \frac{\partial f_{n-1}^m}{\partial z} - \left(\frac{n+m+1}{2n+3}\right) \mathrm{v} \frac{\partial f_{n+1}^m}{\partial z}$$

$$-\frac{\mathrm{v}}{2}\left\{ \begin{aligned} &\frac{1}{2n-1}\left[r^{m-1}\frac{\partial (r^{-m+1} f_{n-1}^{m-1})}{\partial r} - (n-m)(n-m-1) r^{-m-1}\frac{\partial (r^{m+1} f_{n-1}^{m+1})}{\partial r} \right] \\ &+\frac{1}{2n+3}\left[-r^{m-1}\frac{\partial (r^{-m+1} f_{n+1}^{m-1})}{\partial r} + (n+m+1)(n+m+2) r^{-m-1}\frac{\partial (r^{m+1} f_{n+1}^{m+1})}{\partial r} \right] \end{aligned} \right\}$$

Advection in momentum due to electric field

$$\frac{\partial f_n^m}{\partial t} = -eE_z \left\{ \frac{n-m}{2n-1} G_{n-1}^m + \frac{n+m+1}{2n+3} H_{n+1}^m \right\}$$

$$-eE_r \frac{1}{2} \left\{ \frac{1}{2n-1} \left[G_{n-1}^{m-1} - (n-m)(n-m-1) G_{n-1}^{m+1} \right] + \frac{1}{2n+3} \left[-H_{n+1}^{m-1} + (n+m+1)(n+m+2) H_{n+1}^{m+1} \right] \right\}$$

$$-eE_\vartheta \frac{i}{2} \left\{ \frac{1}{2n-1} \left[-G_{n-1}^{m-1} - (n-m)(n-m-1) G_{n-1}^{m+1} \right] + \frac{1}{2n+3} \left[H_{n+1}^{m-1} + (n+m+1)(n+m+2) H_{n+1}^{m+1} \right] \right\}$$

$$G_n^m(p) = \frac{\partial f_n^m}{\partial p} - n \frac{f_n^m}{p} = p^n \frac{\partial (p^{-n} f_n^m)}{\partial p}$$

$$H_n^m(p) = \frac{\partial f_n^m}{\partial p} + (n+1) \frac{f_n^m}{p} = \frac{1}{p^{n+1}} \frac{\partial (p^{n+1} f_n^m)}{\partial p}$$

Rotation by magnetic field: $\omega = eB/\gamma m$

$$\frac{\partial f_n^m}{\partial t} = \omega_z i \left\{ m f_n^m \right\}$$

$$+ \omega_r \frac{i}{2} \left\{ (n-m)(n+m+1) f_n^{m+1} + f_n^{m-1} \right\}$$

$$+ \omega_\vartheta \frac{1}{2} \left\{ (n-m)(n+m+1) f_n^{m+1} - f_n^{m-1} \right\}$$

Collisions

1) Scattering by ions
2) Scattering by isotropic electron distribution

$$\frac{\partial f_n^m}{\partial t} = -\frac{n(n+1)}{2} \upsilon_\perp f_n^m + \frac{\upsilon_\parallel}{v^2} \frac{\partial}{\partial v}\left(D_\parallel \frac{\partial f_n^m}{\partial v} + E f_n^m \right)$$

Assumes Rosenbluth potentials dominated by f_0^0

$D_\parallel$ & *E* are integrals over f_0^0 in velocity space

ADDITIONALLY
Momentum conservation requires scattering by f_1^m

Implicit & explicit in KALOS

$$\frac{\partial f}{\partial t} + \mathbf{v}.\frac{\partial f}{\partial \mathbf{r}} - e(\mathbf{E} + \mathbf{v}\times\mathbf{B}).\frac{\partial f}{\partial \mathbf{p}} = C(f) \qquad \frac{\partial \mathbf{E}}{\partial t} = -\frac{\mathbf{j}}{\varepsilon_0} + c^2\nabla\times\mathbf{B} \qquad \frac{\partial \mathbf{B}}{\partial t} = -\nabla\times\mathbf{E}$$

Implicit

- Calculation of electric field $\frac{\partial \mathbf{E}}{\partial t} = -\frac{\mathbf{j}}{\varepsilon_0}$ ← maintain neutrality
- Collisions $\frac{\partial f}{\partial t} = C(f)$
- Rotation by magnetic field $\frac{\partial f}{\partial t} = e(\mathbf{v}\times\mathbf{B}).\frac{\partial f}{\partial \mathbf{p}}$

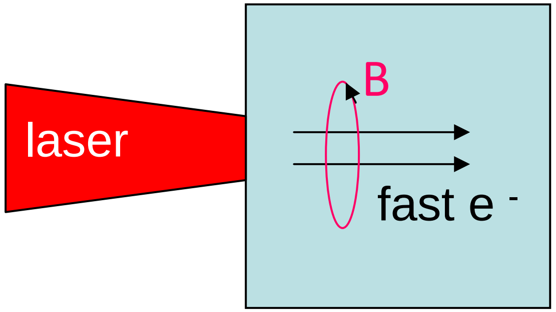
Implicit because short timescale (applies to the electric field calculation, collisions and magnetic rotation)

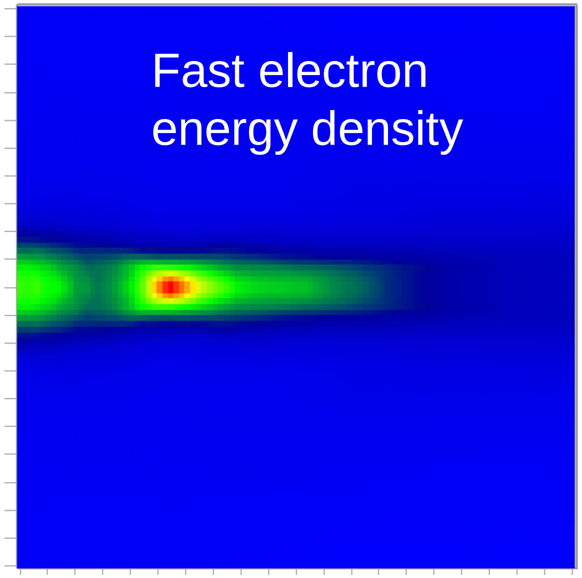
Explicit

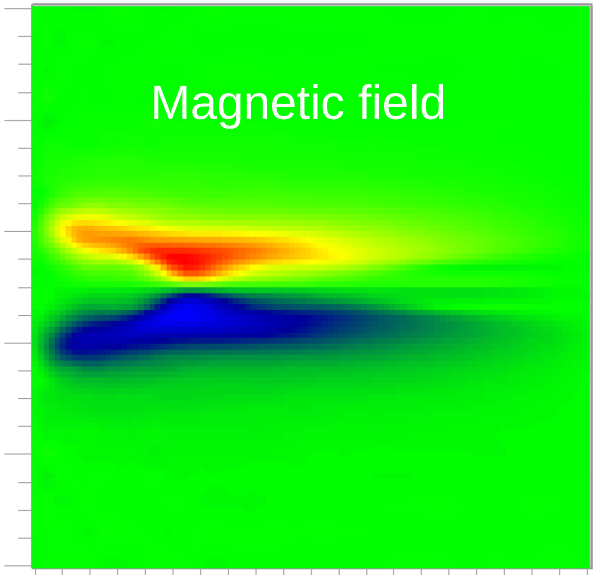
- Advection $\frac{\partial f}{\partial t} = -\mathbf{v}.\frac{\partial f}{\partial \mathbf{r}}$
- Acceleration by electric field $\frac{\partial f}{\partial t} = e\mathbf{E}.\frac{\partial f}{\partial \mathbf{p}}$
- Calculation of magnetic field $\frac{\partial \mathbf{B}}{\partial t} = -\nabla\times\mathbf{E}$

Fast electron collimation
by self-generated magnetic field

Bell & Kingham 2003







A new VFP code written by Michail Tzoufras

Based on spherical harmonics

A Vlasov-Fokker-Planck code for high energy density physics

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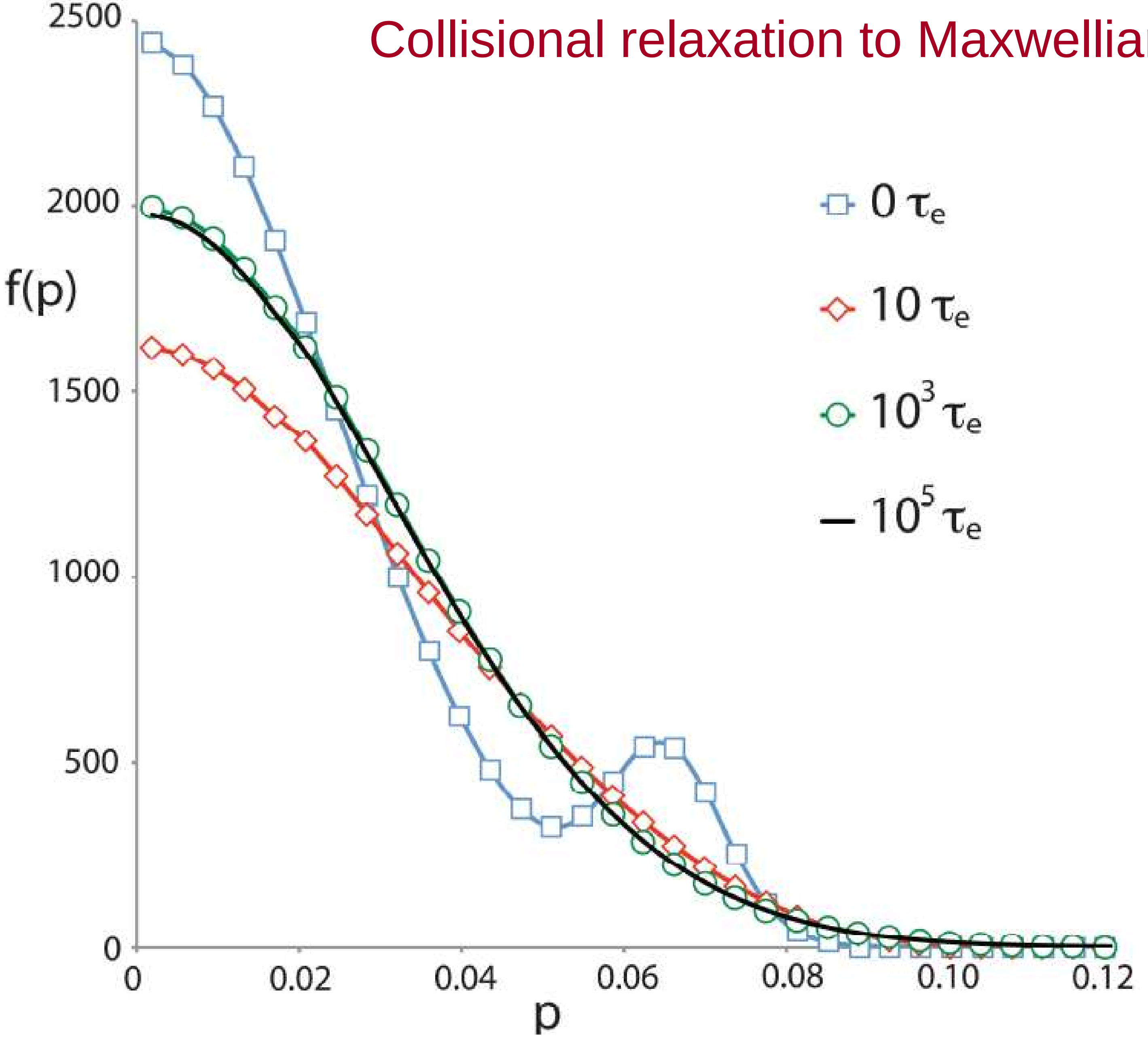
4th order Runge-Kutta
Collisions conservative in number and energy
Full f_0, f_1 collision interaction

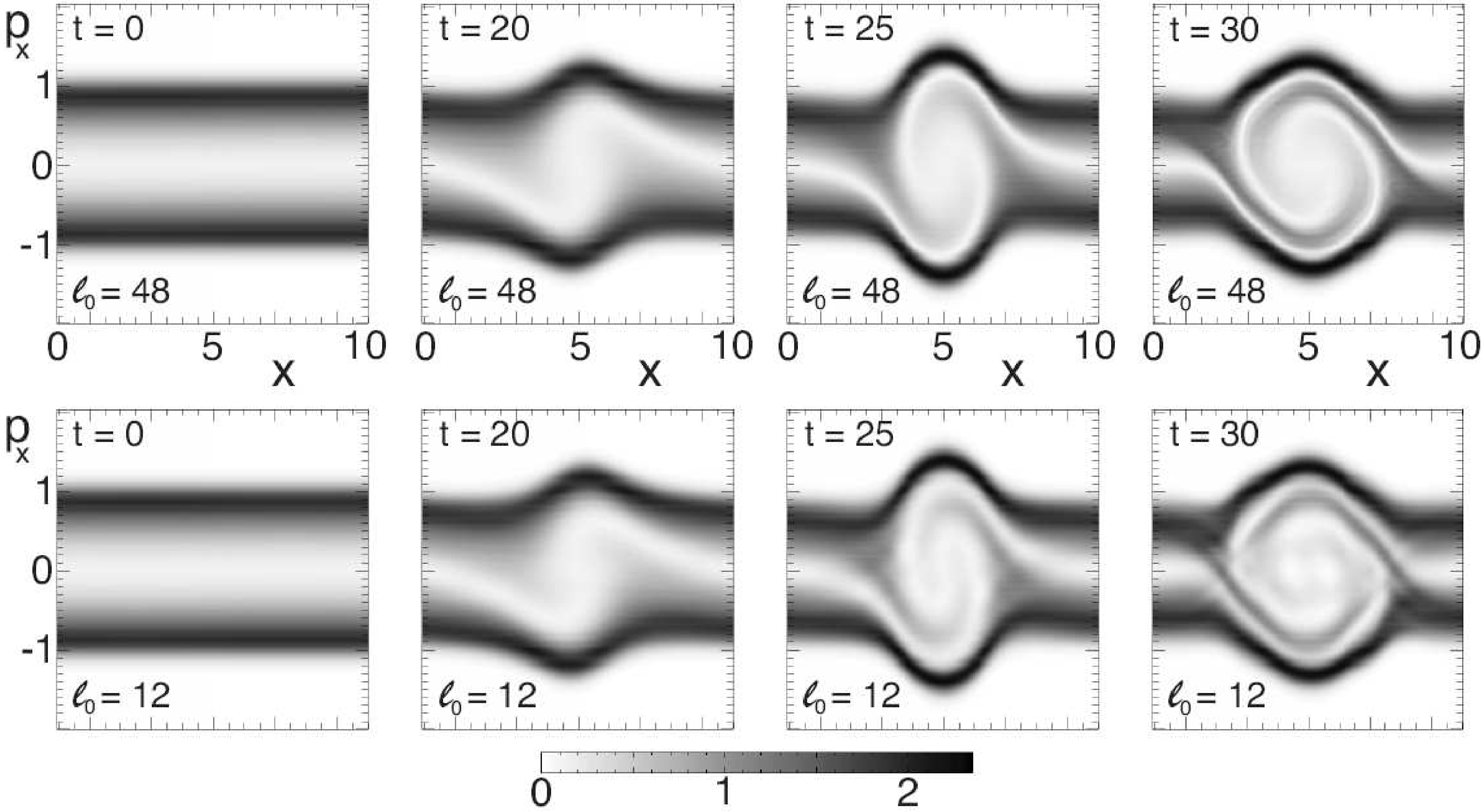
Collisional relaxation to Maxwellian



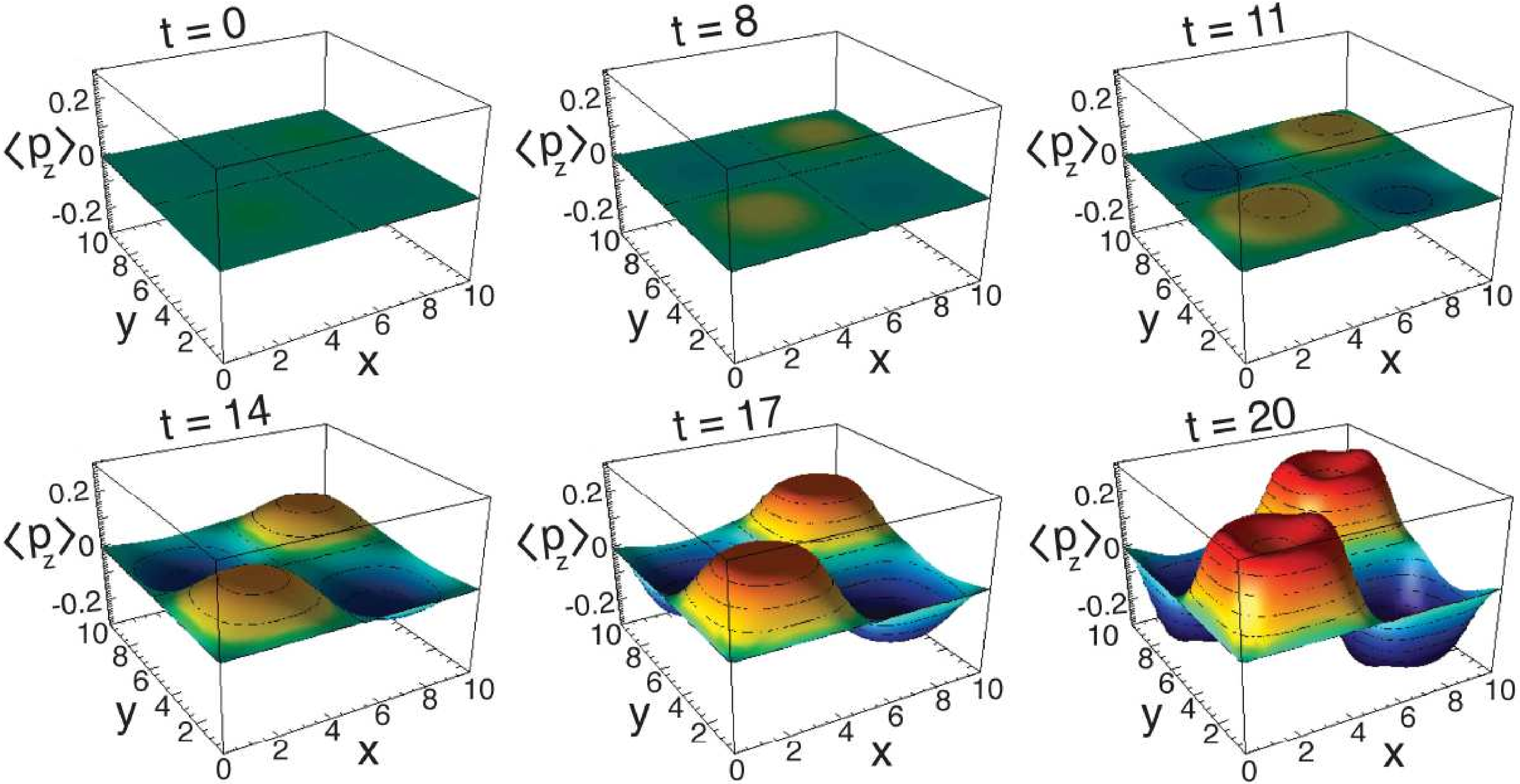
Reproduces Spitzer

Spitzer-Härm heat conduction	$Z = 1$ $\kappa = 3.20$	$Z = 2$ $\kappa = 4.96$	$Z = 4$ $\kappa = 6.98$	$Z = 16$ $\kappa = 10.63$
Equation (39)	3.21	4.93	7.00	10.82*
Tri-diagonal terms	3.04	4.75	6.81	10.71*

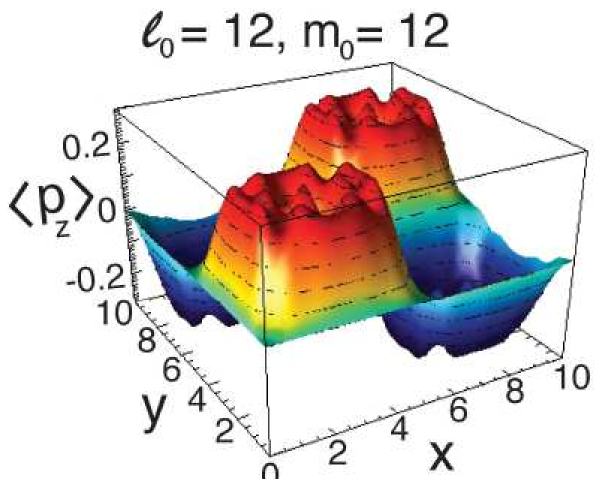
Two-stream instability



Weibel instability



Weibel instability



Conclusion

VFP models kinetic phenomena on hydro timescales

Best method when particles are diffusive but not in thermal equilibrium

Recent technical advances increase range of application
from collisionless to collision-dominated